Homework V First-Half

Due in class June 27 2017

0. Read the following Sections:

Chapter 11 Functions of Several Variables: Section 11.3 Geometric Representation, 11.5 Functions of More Variables, 11.6 Partial Derivatives with More Variables, 11.7 Economic Applications, 11.8 Partial Elasticities

Chapter 12 Tools for Comparative Statics: 12.3 Implicit Differentiation Along A Level Curve, 12.4 More General Cases, 12.5 Elasiticity of Substitution

1. Find the partial elasticities of z with respect to x and y when

$$z = \frac{1}{\sqrt{x^2 + y^2}}$$

2. Let x and y be the population of two cities and d the distance between them. Suppose that the number of travelers T between the cities is given by

$$T = \frac{kxy}{d^n} \quad (k \text{ and } n \text{ are positive constants})$$

Find $\frac{\partial T}{\partial x}$, $\frac{\partial T}{\partial y}$ and $\frac{\partial T}{\partial d}$, and discuss their signs.

3. Find all the first and second order derivatives of

$$w = 3xyz + x^2y - xz^3$$

(You may apply the Young's Theorem)

4. Draw three level curves of the function

$$f(x, y) = x^2 + y^2$$

5. Construct an example of $z = F(x, y)$ such that the level set of

$$F(x, y) = 0$$

   is the union of the x-axis and y-axis.

6. Find the tangent line of $3x^2 - 3xy^2 + y^3 + 3y^2 = 4$ at point $(1, 1)$, using implicit differentiation along a level curve.

7. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ where z is implicitly defined as a function of x and y by $x^3 + y^3 + z^3 - 3z = 0$

8. Find the marginal rate of substitution of y for x in

$$F(x, y) = x^a + y^a$$

   where a is a constant, $a \neq 0$ and $a \neq 1$.

9. Compute the Elasticity of Substitution between y and x in the previous question.