Homework IV Second-Half

Due in class June 20 2017

0. Read the following Sections:
Chapter 11 Functions of Several Variables: Section 11.1 Functions of Two Variables, 11.2 Partial Derivatives with Two Variables
Chapter 12 Tools for Comparative Statics: Section 12.1 A Simple Chain Rule, 12.2 Chain Rule for Many Variables

1. Find the domain of the function $f(x,y) = \ln(x-1) + \sqrt{4-x^2-y^2}$ and draw the set in the coordinate plane
2. Find the domain of the function $f(x) = \frac{1}{e^{x+y}-1}$
3. Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions:
   (a). $f(x,y) = x^8 e^{3x}$
   (b). $f(x,y) = \frac{\ln x}{\sqrt{y^2+1}}$
4. Compute $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial y^2}$ of the following functions:
   (a). $f(x,y) = \frac{x-y}{x+y}$
   (b). $f(x,y) = \sqrt{x^2+y^2}$
   (c). $f(x,y) = y \ln x$
5. Find $\frac{dz}{dt}$ where $z = F(x,y) = x^p y^q$, $x = at$, $y = bt$
6. Find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ where $z = F(x,y) = 2x^2 + 3y^2$, $x = t^2 - s$, $y = t + 2s^3$
7. Find $\frac{\partial z}{\partial t}(1,0)$ where $z = F(x,y) = e^{x-y} + \ln(x+y)$, $x = t+s$, $y = t-s$

8. A particle is moving on the coordinate plane. At position (x, y), its distance to the origin is given by the function

$$D = D(x, y) = \sqrt{x^2 + y^2}$$

Since the particle is moving, its position is a function of time, i.e. $x = f(t)$ and $y = g(t)$. Then the distance of this particle to the origin is also a function of time:

$$D = D(f(t), g(t))$$

The radial velocity is defined to be

$$V_r(t) = \frac{dD}{dt}$$

If it is given that $x = f(t) = t^2$ and $y = g(t) = t^3 + t$, compute the radial velocity at time $t = 2$, i.e. $V_r(2)$.

Remark. The radial velocity has important applications in astronomy. If you are interested, you may read the corresponding Wikipedia page: `https://en.wikipedia.org/wiki/Radial_velocity`