1. Compute the limit
$$\lim_{z\to\infty}\frac{3iz+1}{2z-5i}$$
Solution:
$$\begin{aligned}\lim_{z\to\infty}\frac{3iz+1}{2z-5i} &= \lim_{z\to 0}\frac{\frac{3i}{z}+1}{\frac{2}{z}-5i}\\ &= \lim_{z\to 0}\frac{3i+z}{2-5iz}\\ &= \frac{3i}{2}\end{aligned}$$

2. Show that $\lim_{z\to 0}(\frac{z}{\bar{z}})^2$ doesn't exist, by first considering z approaching 0 along real-axis, then along the line $y = x$.
Solution: If $z \to 0$ along x-axis, $z = x$, so
$$\lim_{x\to 0}(\frac{x}{x})^2 = \lim_{x\to 0} 1^2 = 1$$
If $z \to 0$ along $y = x$, $z = x + xi$, so
$$\lim_{x\to 0}(\frac{x+xi}{x-xi})^2 = \lim_{\frac{1+i}{1-i}} 1^2 = -1$$
So $\lim_{z\to 0}(\frac{z}{\bar{z}})^2$ doesn't exist.

3. $f(z) = z^2$. Compute $f'(z)$ by definition.
Solution:
$$\begin{aligned}f'(z) &= \lim_{\Delta z\to 0}\frac{f(z+\Delta z)-f(z)}{\Delta z}\\ &= \lim_{\Delta z\to 0}\frac{(z+\Delta z)^2-z^2}{\Delta z}\\ &= \lim_{\Delta z\to 0}\frac{2z\Delta z+(\Delta z)^2}{\Delta z}\\ &= \lim_{\Delta z\to 0} 2z+\Delta z\\ &= 2z\end{aligned}$$

4. Show that $f(z) = Re(z)$, i.e. the function sending each complex number to its real part, is not differentiable at any point $z \in \mathbb{C}$.

   Solution: We need to show $f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}$ doesn't exist.

   If $\Delta z \to 0$ along x-axis, $\Delta z = \Delta x$, so

$$\lim_{\Delta x \to 0} \frac{Re(z + \Delta x) - Re(z)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = 1$$

   If $\Delta z \to 0$ along y-axis, $\Delta z = \Delta yi$, so

$$\lim_{\Delta yi \to 0} \frac{Re(z + \Delta yi) - Re(z)}{\Delta x} = \lim_{\Delta yi \to 0} \frac{0}{\Delta x} = 0$$

   We conclude $f'(z)$ doesn't exist at any point.

5. f is a complex function defined in a neighbourhood of $z \in \mathbb{C}$. If f is differentiable at z, prove it is continuous at z.

   Solution: We need to show $\lim_{\Delta z \to 0} f(z + \Delta z) = f(z)$.

   If $f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} = 0$: For any $\epsilon > 0$, there exists $\delta' > 0$ such that $0 < |\Delta z| < \delta'$ implies $|\frac{f(z+\Delta z)-f(z)}{\Delta z}| < \sqrt{\epsilon}$. Take $\delta = \min\{\delta', \sqrt{\epsilon}\}$, we get for any $0 < |\Delta z| < \delta$, $|f(z + \Delta z) - f(z)| < |\Delta z|\sqrt{\epsilon} < \epsilon$.

   If $f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} \neq 0$: For any $\epsilon > 0$, there exists $\delta' > 0$ such that $0 < |\Delta z| < \delta'$ implies $|\frac{f(z+\Delta z)-f(z)}{\Delta z} - f'(z)| < |f'(z)|$. Take $\delta = \frac{\epsilon}{2|f'(z)|}$, we get for any $0 < |\Delta z| < \delta$, $|f(z + \Delta z) - f(z)| < |f'(z) + f'(z)||\Delta z| < 2|f'(z)||\Delta z| < \epsilon$.