1. Compute the limit
\[ \lim_{z \to \infty} \frac{3iz + 1}{2z - 5i} \]

**Solution:**
\[
\lim_{z \to \infty} \frac{3iz + 1}{2z - 5i} = \lim_{z \to 0} \frac{\frac{3i}{z} + \frac{1}{z^2}}{\frac{2}{z} - \frac{5i}{z^2}} = \lim_{z \to 0} \frac{3i + z}{2 - 5iz} = \frac{3i}{2}
\]

2. Show that \( \lim_{z \to 0} (\frac{\bar{z}}{z})^2 \) doesn’t exist, by first considering \( z \) approaching 0 along real-axis, then along the line \( y = x \).

**Solution:** If \( z \to 0 \) along \( x \)-axis, \( z = x \), so
\[
\lim_{x \to 0} \left(\frac{x}{x}\right)^2 = \lim_{x \to 0} 1^2 = 1
\]
If \( z \to 0 \) along \( y = x \), \( z = x + xi \), so
\[
\lim_{x \to 0} \left(\frac{x + xi}{x - xi}\right)^2 = \lim_{x \to 0} 1^2 = -1
\]
So \( \lim_{z \to 0} (\frac{\bar{z}}{z})^2 \) doesn’t exist.

3. \( f(z) = z^2 \). Compute \( f'(z) \) by definition.

**Solution:**
\[
f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} = \lim_{\Delta z \to 0} \frac{(z + \Delta z)^2 - z^2}{\Delta z} = \lim_{\Delta z \to 0} \frac{2z\Delta z + (\Delta z)^2}{\Delta z} = \lim_{\Delta z \to 0} 2z + \Delta z = 2z
\]
4. Show that \( f(z) = Re(z) \), i.e. the function sending each complex number to its real part, is not differentiable at any point \( z \in \mathbb{C} \).

**Solution:** We need to show \( f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} \) doesn’t exist.

If \( \Delta z \to 0 \) along \( x \)-axis, \( \Delta z = \Delta x \), so
\[
\lim_{\Delta x \to 0} \frac{Re(z + \Delta x) - Re(z)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = 1
\]

If \( \Delta z \to 0 \) along \( y \)-axis, \( \Delta z = \Delta yi \), so
\[
\lim_{\Delta yi \to 0} \frac{Re(z + \Delta yi) - Re(z)}{\Delta x} = \lim_{\Delta yi \to 0} \frac{0}{\Delta x} = 0
\]

We conclude \( f'(z) \) doesn’t exist at any point.

5. \( f \) is a complex function defined in a neighbourhood of \( z \in \mathbb{C} \). If \( f \) is differentiable at \( z \), prove it is continuous at \( z \).

**Solution:** We need to show \( \lim_{\Delta z \to 0} f(z + \Delta z) = f(z) \).

If \( f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} = 0 \): For any \( \epsilon > 0 \), there exists \( \delta' > 0 \) such that \( 0 < |\Delta z| < \delta' \) implies \( |\frac{f(z + \Delta z) - f(z)}{\Delta z}| < \sqrt{\epsilon} \). Take \( \delta = \min\{\delta', \sqrt{\epsilon}\} \), we get for any \( 0 < |\Delta z| < \delta \), \( |f(z + \Delta z) - f(z)| < |\Delta z|\sqrt{\epsilon} < \epsilon \).

If \( f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} \neq 0 \): For any \( \epsilon > 0 \), there exists \( \delta' > 0 \) such that \( 0 < |\Delta z| < \delta' \) implies \( |\frac{f(z + \Delta z) - f(z)}{\Delta z} - f'(z)| < |f'(z)| \). Take \( \delta = \frac{\epsilon}{2|f'(z)|} \), we get for any \( 0 < |\Delta z| < \delta \), \( |f(z + \Delta z) - f(z)| < |f'(z) + f'(z)||\Delta z| < 2|f'(z)||\Delta z| < \epsilon \).