1. Compute $\frac{3i+5}{4-3i}$

   Solution:

$$\frac{3i+5}{4-3i} = \frac{(3i+5)(4+3i)}{(4-3i)(4+3i)} = \frac{11+27i}{25}$$

2. $z \in \mathbb{C}$. Prove that $z \in \mathbb{R}$ if and only if $z = \bar{z}$.

   Solution:

   If $z \in \mathbb{R}$, $\bar{z} = \overline{z+0i} = z - 0i = z$.

   If $z \notin \mathbb{R}$, then $z = x + yi$ for some $x, y \in \mathbb{R}$ and $y \neq 0$. Then $\overline{z} = x - yi \neq x + yi = z$.

3. $z_1, z_2 \in \mathbb{C}$. Prove that $z_1 z_2 = 0$ if and only if at least one of z_1, z_2 is 0.

   Solution:

   If $z_1 = 0$ or $z_2 = 0$, it is obvious $z_1 z_2 = 0$.

   If $z_1 \neq 0$ and $z_2 \neq 0$, we can write them in exponential form: $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$ such that $r_1 > 0, r_2 > 0$.

   $z_1 z_2 = r_1 r_2 e^{i(\theta_1 \theta_2)}$, and $r_1 r_2 > 0$, so $z_1 z_2 \neq 0$.

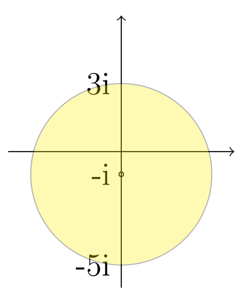
4. If $z = x + yi$, where $x, y \in \mathbb{R}$, prove that $\sqrt{2}|z| \geq |x| + |y|$

   Solution: $|z|^2 = x^2 + y^2 = |x|^2 + |y|^2$, so

   $2|z|^2 = (|x|^2 + |y|^2) + (|x|^2 + |y|^2) \geq |x|^2 + |y|^2 + 2|x||y| = (|x| + |y|)^2$. Taking Square roots, we get $\sqrt{2}|z| \geq |x| + |y|$.

5. Sketch the region in the complex plane described by $|z + i| \leq 4$.

   Solution:



6. Compute $(1+\sqrt{3}i)^6$

   Solution:

$$(1+\sqrt{3}i)^6 = (2e^{\frac{\pi}{3}i})^6 = 2^6(e^{\frac{\pi}{3}i})^6 = 64e^{6\times(\frac{\pi}{3})i} = 64e^{2\pi i} = 64$$

7. $z_1, z_2 \in \mathbb{C}$. Show that if $Re(z_1) > 0$ and $Re(z_2) > 0$, then

$$Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$$

   Solution: If $Re(z_1) > 0$ and $Re(z_2) > 0$, then

$$\begin{cases} -\frac{\pi}{2} < Arg(z_1) < \frac{\pi}{2} \\ -\frac{\pi}{2} < Arg(z_2) < \frac{\pi}{2} \end{cases}$$

   So $-\pi < Arg(z_1) + Arg(z_2) < \pi$.

   $Arg(z_1 z_2) \in \{Arg(z_1)+Arg(z_2)+2\pi k \in \mathbb{R} | k \in \mathbb{Z}\} \cap (-\pi, \pi]$, we get $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$

8. Find all the complex solutions of the equation $z^4 + 4 = 0$.

   Solution:

   $z^4 = -4 = 4e^{\pi + 2k\pi}$, $k \in \mathbb{Z}$.

   So the solutions are

   $\{\sqrt[4]{4}e^{(\frac{\pi+2k\pi}{4})i} \in \mathbb{C} | k = 0,1,2,3\} = \{\sqrt{2}e^{(\frac{\pi}{4}+\frac{k\pi}{2})i} | k = 0,1,2,3\} = \{\sqrt{2}e^{\frac{\pi}{4}}, \sqrt{2}e^{\frac{3\pi}{4}}, \sqrt{2}e^{\frac{5\pi}{4}}, \sqrt{2}e^{\frac{7\pi}{4}}\} = \{1+i, -1+i, -1-i, 1-i\}$