

1. Are the following series convergent? Why?
  - (i).  $\sum_{n=1}^{\infty} e^{in}$
  - (ii).  $\sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{i}{n}$
  - (iii).  $\sum_{n=1}^{\infty} \frac{e^{in}}{n^2}$
2. If  $\lim_{n \rightarrow \infty} z_n = z$ , prove  $\lim_{n \rightarrow \infty} |z_n| = |z|$ .
3. If  $\sum_{n=1}^{\infty} z_n = z$ , prove  $\sum_{n=1}^{\infty} \bar{z}_n = \bar{z}$
4. Find the Taylor series expansion of  $f(z) = \frac{1}{z}$  at  $z_0 = 2$ .
5. Find the Taylor series expansion of  $f(z) = \frac{z}{z^4+4}$  at  $z_0 = 0$ .
6. Consider  $f(z) = \log z$  in the branch  $a < \theta < a + 2\pi$ , where  $a$  is a constant real number such that  $a \notin 2\pi\mathbb{Z}$ . Find the Taylor series expansion of  $f(z)$  at  $z_0 = 1$ .