1. C is the circle $|z| = 2$, positively oriented. Compute

$$\int_C \frac{1}{z^2+1}\,dz$$

2. $f(z) = u(x, y) + iv(x, y)$ is entire. If there exists $u_0 \in \mathbb{R}$ such that $u(x, y) \leq u_0$ for all x, y, prove $u(x, y)$ is a constant function. [Hint: consider $e^{f(z)}$]

3. $p(z) \in \mathbb{C}[z]$ is a non-constant polynomial. c is a root of $p(z)$. Prove:

   (i). $z^k - c^k = (z - c)(z^{k-1} + z^{k-2}c + ... + zc^{k-2} + c^{k-1})$

   (ii). Making use of (i) to prove there exists a polynomial $q(z)$ such that

$$p(z) = p(z) - p(c) = (z - c)q(z)$$

4. (i). C is the circle $|z| = 1$. Prove $g(z) = \frac{z-1}{z+1}$ maps $U = \{x + iy \in \mathbb{C}|x > 0\}$ to the interior of C.

   (ii). f is an entire function. L is a straight line on $\mathbb{C}$. If the image of f all lie on the same side of L, prove f is a constant function.

   (iii). Prove Question (2) again using Question 4(ii).