1. $C$ is the circle $|z| = 2$, positively oriented. Compute
\[ \int_C \frac{1}{z^2 + 1} \, dz \]

2. $f(z) = u(x,y) + iv(x,y)$ is entire. If there exists $u_0 \in \mathbb{R}$ such that $u(x,y) \leq u_0$ for all $x,y$, prove $u(x,y)$ is a constant function. [Hint: consider $e^{f(z)}$]

3. $p(z) \in \mathbb{C}[z]$ is a non-constant polynomial. $c$ is a root of $p(z)$. Prove:
   (i). $z^k - c^k = (z - c)(z^{k-1} + z^{k-2}c + \ldots + zc^{k-2} + c^{k-1})$
   (ii). Making use of (i) to prove there exists a polynomial $q(z)$ such that
   \[ p(z) = p(z) - p(c) = (z - c)q(z) \]

4. (i). $C$ is the circle $|z| = 1$. Prove $g(z) = \frac{z-1}{z+1}$ maps $U = \{x + iy \in \mathbb{C} | x > 0 \}$ to the interior of $C$.
   (ii). $f$ is an entire function. $L$ is a straight line on $\mathbb{C}$. If the image of $f$ all lie on the same side of $L$, prove $f$ is a constant function.
   (iii). Prove Question (2) again using Question 4(ii).