

1. Prove  $\int_C \operatorname{Log}(z+2) dz = 0$ , where  $C$  is the positively oriented unit circle  $|z| = 1$
2. Let  $C_1$  denote the positively oriented boundary of the square whose sides lie along the lines  $x = \pm 1$ ,  $y = \pm 1$  and let  $C_2$  be the positively oriented circle  $|z| = 4$ . Prove

$$\int_{C_1} \frac{z}{1 - e^z} dz = \int_{C_2} \frac{z}{1 - e^z} dz$$

3. Show that if  $C$  is a positively oriented simple closed contour, then the area of the region enclosed by  $C$  is

$$\frac{1}{2i} \int_C \bar{z} dz$$

4. Let  $C$  be the circle  $|z| = 3$  positively oriented. If the function  $g(z)$  ( $|z| \neq 3$ ) is defined to be

$$g(z) = \int_C \frac{2s^2 - s - 2}{s - z} ds$$

Compute  $g(2)$  and  $g(4)$ .

5. Let  $C$  be the unit circle  $z = e^{i\theta}$ ,  $-\pi \leq \theta \leq \pi$ .

(i). Show that for any real constant  $a$ ,  $\int_C \frac{e^{az}}{z} dz = 2\pi i$

(ii). Write the above integral in terms of  $\theta$  to prove

$$\int_0^\pi e^{a \cos \theta} \cos(a \sin \theta) d\theta = \pi$$

6. Let  $f$  be an entire function such that  $|f(z)| \leq A|z|$  for all  $z$ , where  $A$  is a fixed positive number. Show that  $f(z) = az$ , where  $a$  is a complex constant.