

1. Prove $\int_C \text{Log}(z+2) dz = 0$, where C is the positively oriented unit circle $|z| = 1$
2. Let C_1 denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 1$, $y = \pm 1$ and let C_2 be the positively oriented circle $|z| = 4$. Prove

$$\int_{C_1} \frac{z}{1-e^z} dz = \int_{C_2} \frac{z}{1-e^z} dz$$

3. Show that if C is a positively oriented simple closed contour, then the area of the region enclosed by C is

$$\frac{1}{2i} \int_C \bar{z} dz$$

4. Let C be the circle $|z| = 3$ positively oriented. If the function $g(z)$ ($|z| \neq 3$) is defined to be

$$g(z) = \int_C \frac{2s^2 - s - 2}{s - z} ds$$

Compute $g(2)$ and $g(4)$.

5. Let C be the unit circle $z = e^{i\theta}$, $-\pi \leq \theta \leq \pi$.
 - (i). Show that for any real constant a , $\int_C \frac{e^{az}}{z} dz = 2\pi i$
 - (ii). Write the above integral in terms of θ to prove

$$\int_0^\pi e^{a \cos \theta} \cos(a \sin \theta) d\theta = \pi$$

6. Let f be an entire function such that $|f(z)| \leq A|z|$ for all z , where A is a fixed positive number. Show that $f(z) = az$, where a is a complex constant.