

1. Prove $\sin(2z) = 2 \sin z \cos z$ for any $z \in \mathbb{C}$.
2. Show that $\overline{\cos z} = \cos \bar{z}$ for any $z \in \mathbb{C}$.
3. Evaluate the integral $\int_0^1 (1 + it)^2 dt$
4. If $k \in \mathbb{Z}$, evaluate $\int_0^{2\pi} e^{ikt} dt$
5. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function and $z(t) : \mathbb{R} \rightarrow \mathbb{C}$ is also a differentiable function, prove $\frac{d}{dt}(z \circ f) = z'(f(t))f'(t)$
6. Prove the arclength of a curve on \mathbb{C} is independent of the parametrization, i.e. If $z(t) = x(t) + y(t)i$, $a \leq t \leq b$ is a differentiable curve, and $t = \phi(\tau) : [c, d] \rightarrow [a, b]$ is a differentiable bijective function, then the arclength of $z(t)$, $a \leq t \leq b$ equals to the arclength of $z(\phi(\tau))$, $c \leq \tau \leq d$.