

1. Prove $|e^{z^2}| \leq e^{|z|^2}$ for any $z \in \mathbb{C}$.
2. Compute the following:
 - (i). $\text{Log}(-ei)$
 - (ii). $\text{Log}(1 - i)$
 - (iii). $\log 1$
3. Show that $\log(i^2) \neq 2\log(i)$ on the branch $\frac{3\pi}{4} < \theta < \frac{11\pi}{4}$
4. Proving $\log(\frac{1}{z}) = -\log z$ as a multi-valued function.
5. Compute $4^{\frac{1}{2}}$ using the definition of complex power function
6. Prove that $f(z) = z^c, (z \neq 0)$ is a single-valued function if and only if $c \in \mathbb{Z}$.
7. (0 Credit, but you are encouraged to do it if you have some knowledge of group theory.)

Prove the set of all complex numbers of norm 1 form a group under multiplication, and this group is isomorphic to $SO_2(\mathbb{R}) = O_2(\mathbb{R}) \cap SL_2(\mathbb{R}) = \{A \in M_{2 \times 2}(\mathbb{R}) | A^T = A^{-1}, \det(A) = 1\}$.