

1. Show that $f'(z)$ doesn't exist at any point if $f(z) = z + \bar{z}$.
2. Determine when $f'(z)$ exists if $f(z) = z \operatorname{Re}(z)$, where $\operatorname{Re}(z)$ denotes the real part of z .
3. Prove that $f(z) = (3x + y) + i(3y - x)$ is entire.
4. (i). $z = x + yi \in \mathbb{C}$, prove $x = \frac{z + \bar{z}}{2}$ and $y = \frac{z - \bar{z}}{2i}$
(ii). We define $\frac{\partial}{\partial \bar{z}} = \frac{1}{2} \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right)$. Prove if $f'(z)$ exists, then $\frac{\partial f}{\partial \bar{z}} = 0$.
5. Derive the polar form of Cauchy-Riemann Equations. (Recall that $x = r \cos \theta$ and $y = r \sin \theta$).
6. Derive the polar form of Laplace's equation $u_{xx} + u_{yy} = 0$.
7. If $u(x, y)$ is a harmonic function on a domain D , we say $v(x, y)$ is a harmonic conjugate of $u(x, y)$ if the complex function $f(z) = u(x, y) + v(x, y)i$ has derivative. Prove that if $v(x, y)$ and $w(x, y)$ are both harmonic conjugates of $u(x, y)$ on D , then $v(x, y)$ and $w(x, y)$ are differed by a constant.