

1. Show that  $f'(z)$  doesn't exist at any point if  $f(z) = z + \bar{z}$ .
2. Determine when  $f'(z)$  exists if  $f(z) = z\operatorname{Re}(z)$ , where  $\operatorname{Re}(z)$  denotes the real part of  $z$ .
3. Prove that  $f(z) = (3x + y) + i(3y - x)$  is entire.
4. (i).  $z = x + yi \in \mathbb{C}$ , prove  $x = \frac{z+\bar{z}}{2}$  and  $y = \frac{z-\bar{z}}{2i}$   
(ii). We define  $\frac{\partial}{\partial \bar{z}} = \frac{1}{2}(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y})$ . Prove if  $f'(z)$  exists, then  $\frac{\partial f}{\partial \bar{z}} = 0$ .
5. Derive the polar form of Cauchy-Riemann Equations. (Recall that  $x = r \cos \theta$  and  $y = r \sin \theta$ ).
6. Derive the polar form of Laplace's equation  $u_{xx} + u_{yy} = 0$ .
7. If  $u(x, y)$  is a harmonic function on a domain  $D$ , we say  $v(x, y)$  is a harmonic conjugate of  $u(x, y)$  if the complex function  $f(z) = u(x, y) + v(x, y)i$  has derivative. Prove that if  $v(x, y)$  and  $w(x, y)$  are both harmonic conjugates of  $u(x, y)$  on  $D$ , then  $v(x, y)$  and  $w(x, y)$  are differed by a constant.