1. $f(z) = \frac{1-e^{2z}}{z^4}$. What is the order of the pole $z = 0$?
2. $f(z)$ is analytic at z_0. $g(z) = \frac{f(z)}{z-z_0}$. What type of singularity is z_0 for the function g?
3. Compute the residue of $\frac{1}{z^2(z+1)^2}$ at $z = 0$.
4. Prove $z = 0$ is a pole for $f(z) = \frac{1}{z(e^z-1)}$ and compute the residue at $z = 0$.
5. Show that $z = 0$ is a simple pole for $f(z) = \frac{1}{\sin z}$, and compute the residue at $z = 0$.
6. p and q are functions that are analytic at z_0, and $p(z_0) \neq 0$, $q(z_0) = 0$. Show that if z_0 is a pole of order m for $f(z) = \frac{p(z)}{q(z)}$, then z_0 is a zero of order m for q.
7. C is the positively oriented circle $|z| = e$. Evaluate

$$\int_C \tan z \, dz$$

8. $q(z)$ is a function analytic at z_0, $q(z_0) = 0$, $q'(z_0) \neq 0$. Show that z_0 is a pole of order 2 of $f(z) = \frac{1}{q(z)^2}$, and prove the residue of f at z_0 is $-\frac{q''(z_0)}{(q'(z_0))^3}$