1. \( f(z) = \frac{1-e^{2z}}{z^2} \). What is the order of the pole \( z = 0 \)?

2. \( f(z) \) is analytic at \( z_0 \). \( g(z) = \frac{f(z)}{z-z_0} \). What type of singularity is \( z_0 \) for the function \( g \)?

3. Compute the residue of \( \frac{1}{z^2(z+1)^2} \) at \( z = 0 \).

4. Prove \( z = 0 \) is a pole for \( f(z) = \frac{1}{z(e^z-1)} \) and compute the residue at \( z = 0 \).

5. Show that \( z = 0 \) is a simple pole for \( f(z) = \frac{1}{\sin z} \), and compute the residue at \( z = 0 \).

6. \( p \) and \( q \) are functions that are analytic at \( z_0 \), and \( p(z_0) \neq 0 \), \( q(z_0) = 0 \). Show that if \( z_0 \) is a pole of order \( m \) for \( f(z) = \frac{p(z)}{q(z)} \), then \( z_0 \) is a zero of order \( m \) for \( q \).

7. \( C \) is the positively oriented circle \( |z| = e \). Evaluate

\[
\int_C \tan z \, dz
\]

8. \( q(z) \) is a function analytic at \( z_0 \), \( q(z_0) = 0 \), \( q'(z_0) \neq 0 \). Show that \( z_0 \) is a pole of order 2 of \( f(z) = \frac{1}{q(z)} \), and prove the residue of \( f \) at \( z_0 \) is \( -\frac{q''(z_0)}{(q'(z_0))^3} \).