1. C is the positively oriented circle $|z| = 3$. Evaluate the following integrals:

   (i). $\int_C \frac{e^{-z}}{z^2}\, dz$

   (ii). $\int_C z^2 e^{\frac{1}{z}}\, dz$

2. C is the positively oriented circle $|z| = 3$. Evaluate

$$\int_C \frac{1}{1+z^2}\, dz$$

   by:

   (i). Using the singular points inside C.

   (ii). Using the residue at infinity.

3. $p(z) = a_0 + a_1 z + ... + a_n z^n$, $a_n \neq 0$. $q(z) = b_0 + b_1 z + ... + b_m z^m$, $b_m \neq 0$. If $m \geq n+2$ and all the roots of $q(z) = 0$ are inside a simple closed contour C, prove

$$\int_C \frac{p(z)}{q(z)}\, dz = 0$$

4. C is a positively oriented simple closed curve. f is a function which is analytic on and outside of C except at finitely many points $z_1, ..., z_n$. Prove

$$\int_C f(z)\, dz = 2\pi i[Res_{z=0}(\frac{1}{z^2} f(\frac{1}{z})) - \sum_{k=1}^{n} Res_{z=z_k} f(z)]$$

5. Prove $f(z) = \frac{\sin z}{z}$ has a removable singularity at $z_0 = 0$, and then extend f to an entire function.