Homework II Second-Half

Due in class July 18 2017

0. Read The Following Sections:

Chapter 11 Partial Derivatives: Section 11.3 Partial Derivatives, 11.4 Tangent Planes and Linear Approximations, 11.5 The Chain Rule, 11.6 Directional Derivatives and the Gradient Vector

1. Compute $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial y^2}$ of the following functions:

   (a). $f(x, y) = \frac{x-y}{x+y}$

   (b). $f(x, y) = \sqrt{x^2 + y^2}$

   (c). $f(x, y) = y \ln x$

2. Find all the first and second order derivatives of

$$w = 3xyz + x^2 y - xz^3$$

3. Find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ where $z = F(x, y) = 2x^2 + 3y^2$, $x = t^2 - s$, $y = t + 2s^3$

4. Find the equation of the tangent plane to the given surface at $(1, 1, 1)$:

$$z = \sqrt{xy}$$

5. Find the differential of the function $z = x^3 \ln y$

6. Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is a function of x and y implicitly defined via

$$3x^2 y + 4y^2 z + 5z^2 x = 6$$

7. A firm is producing two brands of goods. The cost of producing x units of Brand A and y units of brand B is $C = x^3y + 3xy^3$. Originally the firm is producing 1 unit of Brand A and 1 unit of Brand B per day. Now the firm decides to produce 1.1 units of Brand A per day, and keep the daily cost unchanged. Estimate the new production of Brand B per day.

8. A particle is moving on the xy-coordinate plane. At position (x, y), its distance to the origin is given by the function

$$D = D(x, y) = \sqrt{x^2 + y^2}$$

Since the particle is moving, its position is a function of time, i.e. $x = f(t)$ and $y = g(t)$. Then the distance of this particle to the origin is also a function of time:

$$D = D(f(t), g(t))$$

The radial velocity is defined to be

$$V_r(t) = \frac{dD}{dt}$$

If it is given that $x = f(t) = t^2$ and $y = g(t) = t^3 + t$, compute the radial velocity at time $t = 2$, i.e. $V_r(2)$.

Remark. The radial velocity has important applications in astronomy. If you are interested, you may read the corresponding Wikipedia page: `https://en.wikipedia.org/wiki/Radial_velocity`

9. A firm produces $Q = f(L)$ units of a commodity, using L units of labor. Assume $f'(L) > 0$ and $f''(L) < 0$. If the firm gets p dollars per unit produced and pays w dollars for a unit of labor, write dowm the profit function, and find the first order condition for profit maximization at $L^* > 0$. Then by implicit differentiation, examine how changes in p and w affect the optimal choice L^*

10. Find the directions in which the directional derivative of $f(x, y) = x^2 + sinxy$ at the point $(1, 0)$ has the value 1.

11. Find all points at which the direction of fastest *increase* of the function

$$f(x, y) = x^2 + y^2 - 2x - 4y$$

is $< 1, 1 >$

12. Suppose you are climbing a hill whose shape is given by the equation $z = 1000 - 0.005x^2 - 0.01y^2$, where x, y, z are measured in meters. You are standing at a point with coordinates $(60, 40, 966)$. The positive x-axis points east and the positive y-axis points north.

    (a). If you walk due south, will you start to ascend or descend? At what rate?

    (b). If you walk northwest, will you start to ascend or descend? At what rate?

    (c). In which direction is the slope largest? What is the rate of ascend in that direction? At what angle above the horizon does the path in that direction begin?

13. Are there points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?