1. Find a Cartesian equation of the parametric curve \((e^t - 1, e^{2t})\)

Solution:

\[
\begin{aligned}
    x &= e^t - 1 \\
    y &= e^{2t}
\end{aligned}
\]

So

\[y = e^{2t} = (e^t)^2 = (x + 1)^2\]

Note \(x = e^t - 1 > -1\), so the Cartesian equation is

\[y = e^{2t} = (e^t)^2 = (x + 1)^2, x > -1\]

2. Give a parametric equation of the curve \(x^3 = y^2\).

Solution: Let \(y = t^3\), then \(x^3 = (t^3)^2 = t^6\), so \(x = t^2\). The parametric curve can be

\((t^2, t^3)\)

3. Find the area enclosed by the curve \((t^2 - 2t, \sqrt{t})\) and the y-axis.

Solution: The curve intersects y-axis when \(t^2 - 2t = 0\), we get \(t = 0\) or \(t = 2\), which correspond to points \((0, 0)\) and \((0, \sqrt{2})\).

The area is

\[
\int_0^{\sqrt{2}} |x| dy = \int_0^2 -(t^2 - 2t) d\sqrt{t} = \int_0^2 -(t^2 - 2t)\frac{t^{-\frac{1}{2}}}{2} dt = \frac{8}{15} \sqrt{2}
\]

4. Find the length of the loop of the curve \((3t - t^3, 3t^2)\).

Solution: a loop will be formed when at different \(t_1 < t_2\), \((3t_1 - t_1^3, 3t_1^2) = (3t_2 - t_2^3, 3t_2^2)\):

\[
\begin{aligned}
    3t_1 - t_1^3 &= 3t_2 - t_2^3 \\
    3t_1^2 &= 3t_2^2
\end{aligned}
\]

We get \(t_1 = -\sqrt{3}, t_2 = \sqrt{3}\), and the point of intersection is \((0, 9)\).

The tangent vector is \(((3t - t^3)', (3t^2)') = (3 - 3t^2, 6t)\)

The length if the loop is

\[
\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{(3 - 3t^2)^2 + (6t)^2} dt = \int_{-\sqrt{3}}^{\sqrt{3}} 3 + 3t^2 dt = 12\sqrt{3}
\]
5. Find the points of intersection of the two parametric curves \((2t, 1 - 4t)\) and 
\((1 - t, 1 + t^2)\).

**Solution:** Assume at the point of intersection, the first curve is at time \(t_1\) and 
the second curve is at time \(t_2\): \((2t_1, 1 - 4t_1) = (1 - t_2, 1 + t_2^2)\):

\[
\begin{align*}
2t_1 &= 1 - t_2 \\
1 - 4t_1 &= 1 + t_2^2
\end{align*}
\]

so

\[1 - 4t_1 = 1 + (1 - 2t_1)^2\]

which reduces to

\[1 + t_1^2 = 0\]

So the system of equations has no solution, we conclude the two curves do not intersect.

**Remark 0.1.** Alternatively, you can first recover the Cartesian equation for 
the two curves, and then discuss about the intersection of the curves given by 
Cartesian equations.