1. Find a Cartisian equation of the parametric curve $(e^t - 1, e^{2t})$

   Solution:
$$\begin{cases} x = e^t - 1 \\ y = e^{2t} \end{cases}$$
   So
$$y = e^{2t} = (e^t)^2 = (x+1)^2$$
   Note $x = e^t - 1 > -1$, so the Cartesian equation is
$$y = e^{2t} = (e^t)^2 = (x+1)^2, x > -1$$

2. Give a parametric equation of the curve $x^3 = y^2$.

   Solution: Let $y = t^3$, then $x^3 = (t^3)^2 = t^6$, so $x = t^2$. The parametric curve can be
$$(t^2, t^3)$$

3. Find the area enclosed by the curve $(t^2 - 2t, \sqrt{t})$ and the y-axis.

   Solution: The curve intersects y-axis when $t^2 - 2t = 0$, we get $t = 0$ or $t = 2$, which correspond to points $(0, 0)$ and $(0, \sqrt{2})$.

   The area is
$$\int_0^{\sqrt{2}} |x|\, dy = \int_0^2 -(t^2 - 2t)\, d\sqrt{t} = \int_0^2 -(t^2 - 2t)\frac{t^{-\frac{1}{2}}}{2}\, dt = \frac{8}{15}\sqrt{2}$$

4. Find the length of the loop of the curve $(3t - t^3, 3t^2)$.

   Solution: a loop will be formed when at different $t_1 < t_2$, $(3t_1 - t_1^3, 3t_1^2) = (3t_2 - t_2^3, 3t_2^2)$:
$$\begin{cases} 3t_1 - t_1^3 = 3t_2 - t_2^3 \\ 3t_1^2 = 3t_2^2 \end{cases}$$
   We get $t_1 = -\sqrt{3}$, $t_2 = \sqrt{3}$, and the point of intersection is $(0, 9)$.

   The tangent vector is $((3t - t^3)', (3t^2)') = (3 - 3t^2, 6t)$

   The length if the loop is
$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{(3 - 3t^2)^2 + (6t)^2}\, dt = \int_{-\sqrt{3}}^{\sqrt{3}} 3 + 3t^2\, dt = 12\sqrt{3}$$

5. Find the points of intersection of the two parametric curves $(2t, 1-4t)$ and $(1-t, 1+t^2)$.

   Solution: Assume at the point of intersection, the first curve is at time t_1 and the second curve is at time t_2: $(2t_1, 1-4t_1) = (1-t_2, 1+t_2^2)$:

$$\begin{cases} 2t_1 = 1 - t_2 \\ 1 - 4t_1 = 1 + t_2^2 \end{cases}$$

   so

$$1 - 4t_1 = 1 + (1 - 2t_1)^2$$

   which reduces to

$$1 + t_1^2 = 0$$

   So the system fo equations has no solution, we conclude the two curves do not intersect.

   Remark 0.1. Alternatively, you can first recover the Cartesian equation for the two curves, and then discuss about the intersection of the curves given by Cartesian equations.