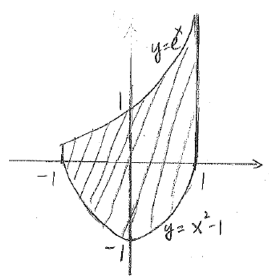
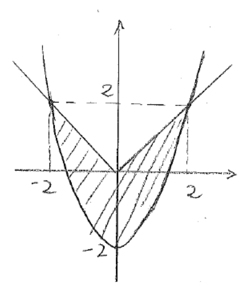
1. Compute the area of the region enclosed by $y = e^x$, $y = x^2 - 1$, $x = -1$ and $x = 1$.

   Solution:

$$\int_{-1}^{1} e^x - (x^2 - 1)\,dx = e^x - \frac{x^3}{3} + x\Big|_{-1}^{1} = \frac{4}{3} - e^{-1} + e$$





2. Compute the area of the the region enclosed by $y = |x|$ and $y = x^2 - 2$.

   Solution:

   Solve

$$\begin{cases} y = |x| \\ y = x^2 - 2 \end{cases}$$

   We get $x = 2, y = 2$ or $x = -2, y = 2$. So the intersection points of these two graphs are $(-2, 2)$ and $(2, 2)$. The area is:

$$\int_{-2}^{2} |x| - (x^2 - 2)\,dx = 2\int_{0}^{2} x - x^2 + 2\,dx = \frac{20}{3}$$

3. Compute the volume of a frustum of a right circular cone with height h, lower base radius R and top radius r.

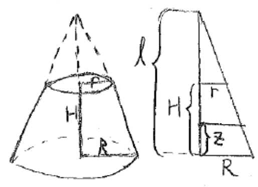
   Solution: We set a vertical axis, and let the lower base at level $z = 0$. Then denote the cross section for level z is a circle of radius ρ. By the similar triangles shown in the picture, we get

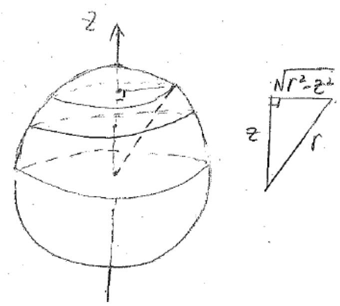
$$\frac{l}{R} = \frac{l - H}{r} = \frac{l - z}{\rho}$$

We get $\rho = R - \frac{R-r}{H} z$.

So the volume is

$$\int_0^H \pi(R - \frac{R-r}{H} z)^2 \, dz = \frac{\pi H(R^2 + r^2 + Rr)}{3}$$





4. Compute the volume of the cap of a sphere with radius r and height h.

   Solution:

   Construct a vertical axis with the centre of the ball at origin. The cross section at level z is $\sqrt{r^2 - z^2}$ by the Pythagorean theorem. So the volume is

$$\int_{r-h}^{r} \pi(r^2 - z^2) \, dz = \pi h^2 (r - \frac{h}{3})$$

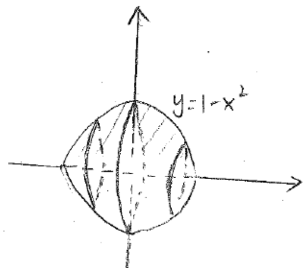
5. Find the volume of the solid obtained by rotating the region bounded by $y = 1 - x^2$ and $y = 0$ about x-axis.

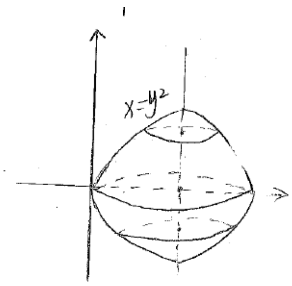
   Solution:

   $y = 1 - x^2$ intersects $y = 0$ at $(-1, 0)$ and $(1, 0)$.

   We take the x-axis and the cross section for each x is a disk of radius $1 - x^2$. So the volume is

$$\int_{-1}^{1} \pi(1 - x^2)^2 \, dx = \frac{16}{15}\pi$$





6. The region enclosed by $x = y^2$, $x = 1$ on the xy-plane is rotated about $x = 1$ to form a solid. Find the volume of the solid.

   Solution:

   The curve $x = y^2$ intersects $x = 1$ at $(1, 1)$ and $(1, -1)$.

   We take the y-axis, and the cross section for each y is $1 - y^2$. So the volume is

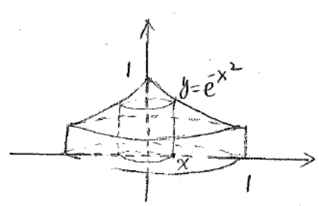
$$\int_{-1}^{1} \pi(1 - y^2)^2 \, dy = \frac{16}{15}\pi$$

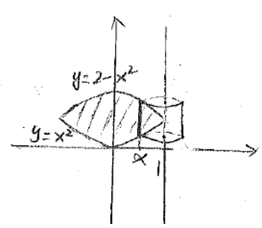
7. Use the cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$ about the y-axis.

   Solution:

   The cylindrical shell of radius x has height e^{-x^2}, so the volume is

$$\int_{0}^{1} 2\pi x e^{-x^2} \, dx = \pi(1 - e^{-1})$$





8. Use the cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 2 - x^2$ about $x = 1$.

   Solution:

$y = x^2$ and $y = 2 - x^2$ intersect at $(-1, 1)$ and $(1, 1)$. The cylindrical shell at x has base radius $1 - x$ and height $(2 - x^2) - x^2 = 2 - 2x^2$, so the volume is

$$\int_{-1}^{1} 2\pi(1 - x)(2 - 2x^2)\, dx = \frac{8\pi}{3}$$