1. Calculate $\int \cos^5 x \sin^8 x \, dx$

   Solution:

$$
\begin{aligned}
&\int \cos^5 x \sin^8 x \, dx \\
=&\int \cos^4 x \sin^8 x \, d \sin x \\
=&\int (1-\sin^2 x)^2 \sin^8 x \, d \sin x \\
=&\int (1-2\sin^2 x+\sin^4 x) \sin^8 x \, d \sin x \\
=&\int \sin^8 x - 2\sin^{10} x + \sin^{12} x \, d \sin x \\
=&\frac{1}{9}\sin^9 x - \frac{2}{11}\sin^{11} x + \frac{1}{13}\sin^{13} x + C
\end{aligned}
$$

2. Calculate $\int \sin^2 x \cos^2 x \, dx$

   Solution:

$$
\begin{aligned}
&\int \sin^2 x \cos^2 x \, dx \\
=&\int (\sin x \cos x)^2 \, dx \\
=&\int (\frac{\sin 2x}{2})^2 \, dx \\
=&\frac{1}{4}\int \sin^2 2x \, dx \\
=&\frac{1}{8}\int \sin^2 2x \, d2x \\
=&\frac{1}{8}\int \frac{1-\cos 4x}{2} \, d2x \\
=&\frac{1}{32}\int 1-\cos 4x \, d4x \\
=&\frac{1}{32}(4x-\sin 4x)+C
\end{aligned}
$$

3. Calculate $\int \frac{1}{\cos x - 1}\, dx$

   Solution:

$$
\begin{aligned}
&\int \frac{1}{\cos x - 1}\, dx \\
=&\int \frac{\cos x + 1}{(\cos x - 1)(\cos x + 1)}\, dx \\
=&\int \frac{\cos x + 1}{\cos^2 x - 1}\, dx \\
=&-\int \frac{\cos x + 1}{\sin^2 x} \\
=&-\int \frac{\cos x}{\sin^2 x}\, dx - \int \frac{1}{\sin^2 x}\, dx \\
=&-\int \frac{1}{\sin^2 x}\, d\sin x + \cot x \\
=&\frac{1}{\sin x} + \frac{\cos x}{\sin x} + C \\
=&\frac{1 + \cos x}{\sin x} + C
\end{aligned}
$$

4. Calculate $\int \frac{1}{\sqrt{9x^2+6x-8}}\,dx$

Solution:

$$\frac{1}{\sqrt{9x^2+6x-8}} = \frac{1}{3\sqrt{x^2+\frac{2}{3}x-\frac{8}{9}}} = \frac{1}{3\sqrt{(x+\frac{1}{3})^2-1}}$$

So we can let $x+\frac{1}{2} = \sec t$, where $0 < t < \frac{\pi}{2}$ or $\pi < t < \frac{3}{2}\pi$

$$\begin{aligned}
\int \frac{1}{\sqrt{9x^2+6x-8}}\,dx &= \int \frac{1}{3\sqrt{(x+\frac{1}{3})^2-1}}\,dx \\
&= \frac{1}{3}\int \frac{1}{\tan t}\,d(\sec t - \frac{1}{3}) \\
&= \frac{1}{3}\int \frac{\cos t}{\sin t}\frac{\sin t}{\cos^2 t}\,dt \\
&= \frac{1}{3}\int \frac{1}{\cos t}\,dt \\
&= \frac{1}{3}\ln(\sec t + \tan t) + C \\
&= \frac{1}{3}\ln(x+\frac{1}{3}+\sqrt{(x+\frac{1}{3})^2-1}) + C
\end{aligned}$$

5. Calculate $\int_0^1 \sqrt{x^2+1}\,dx$

Solution: Let $x = tant$, $u = \sin t$

$$\int_0^1 \sqrt{x^2+1}\,dx = \int_0^{\frac{\pi}{4}} \sqrt{\tan^2 t+1}\,d\tan t = \int_0^{\frac{\pi}{4}} \sec t \times \sec^2 t\,dt$$
$$= \int_0^{\frac{\pi}{4}} \frac{1}{\cos^3 t}\,d\tan t = \int_0^{\frac{\pi}{4}} \frac{\cos t}{\cos^4 t}\,d\tan t = \int_0^{\frac{\pi}{4}} \frac{1}{(1-\sin^2 t)^2}\,d\sin t$$
$$= \int_0^{\frac{\sqrt{2}}{2}} \frac{1}{(1-u^2)^2}\,du = \int_0^{\frac{\sqrt{2}}{2}} \frac{1}{(u-1)^2(u+1)^2}\,du$$

$$\frac{1}{(u-1)^2(u+1)^2}$$
$$=\frac{A}{u-1} + \frac{B}{(u-1)^2} + \frac{C}{u+1} + \frac{D}{(u+1)^2}$$
$$=\frac{(A+C)u^3 + (A+B-C+D)u^2 + (-A+2B-C-2D)u + (-A+B+C+D)}{(u-1)^2(u+1)^2}$$

We get

$$\begin{cases} A+C=0 \\ A+B-C+D=0 \\ -A+2B-C-2D=0 \\ -A+B+C+D=0 \end{cases}$$

Solving the system of equations, we get $A = -\frac{1}{4}$, $B = C = D = \frac{1}{4}$.

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{1}{(u-1)^2(u+1)^2}\,dx = \frac{1}{4}\int_0^{\frac{\sqrt{2}}{2}} \left(\frac{1}{u-1} + \frac{1}{(u-1)^2} + \frac{1}{u+1} + \frac{1}{(u+1)^2}\right)dx$$
$$= \frac{1}{4}\left(-\ln|x-1| - \frac{1}{x-1} + \ln|x+1| - \frac{1}{x+1}\right)\Bigg|_0^{\frac{\sqrt{2}}{2}}$$
$$= \frac{\ln(\sqrt{2}+1)}{2} + 1$$

6. Calculate $\int \sqrt{1-4x^2}\,dx$

   Solution:

   Let $x = \frac{1}{2}\sin t$, where $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$.

$$\begin{aligned}
\int \sqrt{1-4x^2}\,dx &= \int \sqrt{1-4(\frac{1}{2}\sin t)^2}\,d\frac{1}{2}\sin t \\
&= \int \cos t(\frac{\cos t}{2})\,dt \\
&= \frac{1}{2}\int \cos^2 t],dt \\
&= \frac{1}{4}\int 1+\cos 2t\,dt \\
&= \frac{1}{8}\int 1+\cos 2t\,d2t \\
&= \frac{1}{8}(2t+\sin 2t)+C \\
&= \frac{1}{8}(2t+2\sin t\cos t)+C \\
&= \frac{1}{4}(\sin^{-1} 2x + 2x\sqrt{1-4x^2})+C
\end{aligned}$$

7. Calculate $\int \frac{\sqrt{x^2-4}}{x}\, dx$

   Solution:

   Let $x = 2\sec t$, where $0 \leq t < \frac{\pi}{2}$ or $\pi \leq t < \frac{3}{2}\pi$

$$
\begin{aligned}
\int \frac{\sqrt{x^2-4}}{x}\, dx &= \int \frac{2\tan t}{2\sec t}\, d2\sec t \\
&= 2\sin t \times \frac{\sin t}{\cos^2 t}\, dt \\
&= 2\int \tan^2 t\, dt \\
&= 2\int \sec^2 t - 1\, dt \\
&= 2(\tan t - t) + C \\
&= 2(\sqrt{(\frac{x}{2})^2 - 1} - \sec^{-1}(\frac{x}{2})) + C \\
&= 2(\sqrt{(\frac{x}{2})^2 - 1} - \cos^{-1}(\frac{2}{x})) + C \\
&= \sqrt{x^2-4} - 2\cos^{-1}(\frac{2}{x}) + C
\end{aligned}
$$

8. Calculate $\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

   Solution: $x^2 - 5x + 6 = (x-2)(x-3)$.

$$\frac{x-4}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)} = \frac{(A+B)x-(3A+2B)}{(x-2)(x-3)}$$

We get

$$\begin{cases} A+B=1 \\ 3A+2B=4 \end{cases}$$

Solving the equations we obtain $A = 2, B = -1$.

$$\int_0^1 \frac{x-4}{x^2-5x+6}\,dx = \int_0^1 \frac{2}{x-2} - \frac{1}{x-3}\,dx = 3\ln|x-2| - \ln|x-3|\Big|_0^1 = \ln 3 - 3\ln 2$$

9. Calculate $\int \frac{x^5+x-1}{x^3+1}\,dx$

   Solution:

$$\frac{x^5+x-1}{x^3+1} = x^2 + \frac{-x^2+x-1}{x^3+1} = x^2 - \frac{x^2-x+1}{(x+1)(x^2-x+1)} = x^2 - \frac{1}{x+1}$$

So

$$\int \frac{x^5+x-1}{x^3+1}\,dx = \int x^2 - \frac{1}{x+1}\,dx = \frac{1}{3}x^3 - \ln|x+1| + C$$

10. Calculate $\int_9^{16} \frac{\sqrt{x}}{x-4}\, dx$

Solution: Let $u = \sqrt{x}$,

$$\begin{aligned}
\int_9^{16} \frac{\sqrt{x}}{x-4}\, dx &= \int_3^4 \frac{u}{u^2-4} 2u\, du \\
&= 2\int_3^4 \frac{u^2}{u^2-4}\, du \\
&= 2\int_3^4 1 + \frac{1}{u-2} - \frac{1}{u+2}\, du \\
&= 2(u + \ln|u-2| - \ln|u+2|) \\
&= 2(1 + \ln 5 - \ln 3)
\end{aligned}$$