Write your solutions in steps.

1. (3 points) Find the centre of mass of the triangle with vertices at \((-1, 0), (1, 0), (0, 1)\).

2. (4 points) Solve the differential equation:

\[ y' = 2x^2y \]

with initial value \( x = 1, y = 1 \).

3. (3 points) Determine whether the sequence \( \frac{a_{n+3}}{a_n} \) converges or diverges.

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1. The triangle is the region bounded by

\[ f(x) = 1 - 1|x| \text{ and } x\text{-axis} \]

\[ \int_{-1}^{1} f(x) \, dx = \text{area of triangle} = 1 \]

By Symmetric Principle, the centre of mass lies on y-axis, say it’s \((0, y)\). Then

\[ y = \frac{\int_{-1}^{1} \frac{f(x)^2}{2} \, dx}{\int_{-1}^{1} f(x) \, dx} = \frac{\int_{0}^{1} \frac{f(x)^2}{2} \, dx}{\int_{-1}^{1} f(x) \, dx} \]

because \( f(x) \) is an even function

\[ = 2 \int_{0}^{1} \frac{(1-x)^2}{2} \, dx = \frac{1}{3} \]

So centre of mass is \((0, \frac{1}{3})\)

2. \( y' = 2x^2y \)

\[ \frac{dy}{y} = 2x^2 \, dx \]

\[ \int \frac{1}{y} \, dy = \int 2x^2 \, dx \]

\[ \ln |y| = \frac{2}{3} x^3 + C \]

\[ |y| = e^{\frac{2}{3} x^3 + C} \]

\( x = 1, y > 0 \), so \( 1 = e^{\frac{2}{3} + C} \Rightarrow C = \frac{-2}{3} \)

we get \( y = e^{\frac{2}{3} x^3 - \frac{2}{3}} \)

3. \( \lim_{n \to \infty} \frac{a_{n+3}}{a_n} = \lim_{n \to \infty} \left( \frac{\frac{a_{n+3}}{a_n}}{\frac{a_{n+2}}{a_{n+1}}} \right) = 1 \)

\[ = 128 \lim_{n \to \infty} \left( \frac{a_n}{a_1} \right)^n \]

\[ = 128 \times 0 \]

So it converges.