

1. Find a power series representation for the function and determine the interval of convergence:

(i).  $f(x) = \frac{x}{9+x^2}$

(ii).  $f(x) = \frac{1+x}{1-x}$

(iii).  $f(x) = \frac{3}{x^2-x-2}$

(iv).  $f(x) = \frac{x}{(1+4x)^2}$

2. Find the Taylor series of  $f(x) = \ln x$  at 1, and prove  $f(x)$  equals to this Taylor series on  $(\frac{1}{2}, \frac{3}{2})$ .
3. Use binomial series to expand the function  $f(x) = \frac{1}{(2+x)^3}$  as a power series, and state the radius of convergence.
4. Use Taylor series to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{1 + x - e^x}$$

5. Evaluate the integral as as infinite series:

$$\int x \cos(x^3) dx$$

6. Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{4^n}{n5^n}$$