

1. f is a function defined on $[0, 1]$. Some of its values are shown in the table below:

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
$f(x)$	2	2.5	3.5	3	4	4.5	3.5	2	1.5

Approximate $\int_0^1 f(x) dx$ by the following rules:

- (i). Midpoint Rule for $n = 4$
 - (ii). Trapezoid Rule for $n = 4$
 - (iii). Simpson's Rule for $n = 8$
2. How large should n be to guarantee that the Midpoint Rule Approximation to $\int_0^1 e^{x^2} dx$ is accurate to within $0.00001 = 10^{-5}$?
3. Determine if each of the following improper integrals is convergent or divergent, and evaluate if it is convergent:

(i). $\int_3^{+\infty} \frac{1}{(x-2)^{\frac{3}{2}}} dx$

(ii). $\int_0^{+\infty} \frac{x^2}{\sqrt{1+x^3}} dx$

(iii). $\int_{-\infty}^{+\infty} x e^{-x^2} dx$

(iv). $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$

(v). $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$

(vi). $\int_0^9 \frac{1}{\sqrt[3]{x-1}} dx$

(vii). $\int_{-1}^1 \frac{e^x}{e^x-1} dx$

(viii). $\int_0^{+\infty} \frac{1}{\sqrt{x(1+x)}} dx$

4. Show that

$$\int_0^{+\infty} x^2 e^{-x^2} dx = \frac{1}{2} \int_0^{+\infty} e^{-x^2} dx$$