1. \( f \) is a function defined on \([0, 1]\). Some of its values are shown in the table below:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0.125</th>
<th>0.25</th>
<th>0.375</th>
<th>0.5</th>
<th>0.625</th>
<th>0.75</th>
<th>0.875</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>2.5</td>
<td>3.5</td>
<td>3</td>
<td>4</td>
<td>4.5</td>
<td>3.5</td>
<td>2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Approximate \( \int_{0}^{1} f(x) \, dx \) by the following rules:

(i). Midpoint Rule for \( n = 4 \)
(ii). Trapezoid Rule for \( n = 4 \)
(iii). Simpson’s Rule for \( n = 8 \)

2. How large should \( n \) be to guarantee that the Midpoint Rule Approximation to \( \int_{0}^{1} e^{x^2} \, dx \) is accurate to within 0.00001 = 10^{-5}?

3. Determine if each of the following improper integrals is convergent or divergent, and evaluate if it is convergent:

(i). \( \int_{3}^{\infty} \frac{1}{(x-2)^{\frac{3}{2}}} \, dx \)
(ii). \( \int_{0}^{\infty} \frac{x^2}{\sqrt{1+x^3}} \, dx \)
(iii). \( \int_{-\infty}^{\infty} x e^{-x^2} \, dx \)
(iv). \( \int_{0}^{1} \ln x \, dx \)
(v). \( \int_{0}^{1} \frac{1}{\sqrt{1-x^2}} \, dx \)
(vi). \( \int_{0}^{9} \frac{1}{\sqrt{x-1}} \, dx \)
(vii). \( \int_{-1}^{1} \frac{e^x}{e^x-1} \, dx \)
(viii). \( \int_{0}^{\infty} \frac{1}{\sqrt{x}(1+x)} \, dx \)

4. Show that

\[
\int_{0}^{\infty} x^2 e^{-x^2} \, dx = \frac{1}{2} \int_{0}^{\infty} e^{-x^2} \, dx
\]