1. Find the critical numbers of the function $f(x) = |3x - 4|$.

   Solution: When $x > \frac{4}{3}$, $f'(x) = 3$. When $x < \frac{4}{3}$, $f'(x) = -3$. $f'(x)$ doesn't exist at $x = \frac{4}{3}$, so the critical number is $-\frac{4}{3}$.

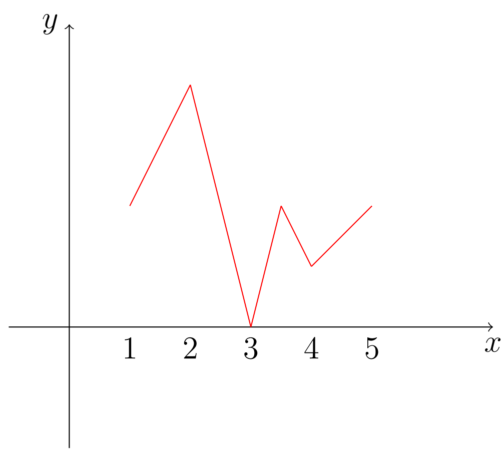
2. Find the absolute maximum and absolution minimum values of the function $f(x) = 5 + 54x - 2x^3$ on the interval $[0, 4]$.

   Solution: $f'(x) = 54 - 6x^2$. Let $f'(x) = 0$, we get $x = 3$ or $x = -3$. So the critical numbers are 3 and -3. Since -3 is not in the interval $[0, 4]$, we only need to consider $x = 3$.

   $f(3) = 149$, $f(0) = 5$ and $f(4) = 93$, so the absolute maximum on $[0, 4]$ is $f(3) = 149$ and the absolute minimum is $f(0) = 5$.

3. Sketch a graph of a function f that is continuous on $[1, 5]$ and has absolute maximum at $x = 2$, absolute minimum at $x = 3$, and local minimum at $x = 4$.

   Solution:



4. Suppose $3 \leq f'(x) \leq 5$ for all values of x. Show that $18 \leq f(8) - f(2) \leq 30$.

   Solution: By the Mean Value Theorem, there is c in $(2, 8)$ such that $f(8) - f(2) = f'(c)(8 - 2) = 6f'(c)$. $3 \leq f'(c) \leq 5$, so $18 \leq 6f'(x) \leq 30$, i.e. $18 \leq f(8) - f(2) \leq 30$.

5. Show that the equation $x^4 + 4x + c = 0$ (c is a constant) has at most two real root.

   Solution: Let $f(x) = x^4 + 4x + c$. Suppose it has at least three roots $x_1 < x_2 < x_3$, then by the Rolle's Theorem, there is a number c on (x_1, x_2) such that $f'(c) = 0$ and there is a number d on (x_2, x_3) such that $f'(d) = 0$.

   But $f'(x) = 4x^3+4 = 4(x^3+1) = 4(x+1)(x^2-x+1) = 4(x+1)((x-\frac{1}{2})^2+\frac{3}{4}) = 0$ has only one solution, so it's impossible that $f'(c) = f'(d) = 0$.

   We conclude f cannot have more than two solutions.

6. Suppose that f and g are continuous on $[a, b]$ and differentiable on (a, b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for all $a < x < b$. Prove that $f(b) < g(b)$.

   Solution: Let $F(x) = f(x) - g(x)$. $F(a) = f(a) - g(a) = 0$ and $F'(x) = f'(x) - g'(x) < 0$ for all $a < x < b$. By the Mean Value Theorem, there is c in (a, b) such that

   $$F(b) - F(a) = F'(c)(b - a)$$

   Since $F(a) = 0$ and $F'(c) < 0$, we get

   $$F(b) = F'(c)(b - a) < 0$$

   So $f(b) - g(b) = F(b) < 0$, i.e. $f(b) < g(b)$.