1. Compute $\lim\limits_{x\to 0} \dfrac{\cos^4 x}{5+x^3}$

   Solution: $\lim\limits_{x\to 0} \dfrac{\cos^4 x}{5+x^3} = \dfrac{\lim\limits_{x\to 0} \cos^4 x}{\lim\limits_{x\to 0}(5+x^3)} = \dfrac{(\lim\limits_{x\to 0} \cos x)^4}{5+\lim\limits_{x\to 0} x^3} = \dfrac{\cos^4 0}{5+0^3} = \dfrac{1}{5}$

2. Compute $\lim\limits_{x\to 5} \dfrac{x^2-6x+5}{x-5}$

   Solution: $\lim\limits_{x\to 5} \dfrac{x^2-6x+5}{x-5} = \lim\limits_{x\to 5} \dfrac{(x-5)(x-1)}{x-5} = \lim\limits_{x\to 5}(x-1) = 5-1 = 4$

3. Compute $\lim\limits_{x\to 3} \dfrac{2x-6}{|x-3|}$

   Solution:
   $$\frac{2x-6}{|x-3|} = \begin{cases} \frac{2x-6}{x-3} = \frac{2(x-3)}{x-3} = 2, x > 3 \\ \frac{2x-6}{3-x} = \frac{2(x-3)}{3-x} = -2, x < 3 \end{cases}$$
   So $\lim\limits_{x\to 3^+} \dfrac{2x-6}{|x-3|} = \lim\limits_{x\to 3^+} 2 = 2$, $\lim\limits_{x\to 3^-} \dfrac{2x-6}{|x-3|} = \lim\limits_{x\to 3^-} -2 = -2$

   We see $\lim\limits_{x\to 3^+} \dfrac{2x-6}{|x-3|} \neq \lim\limits_{x\to 3^-} \dfrac{2x-6}{|x-3|}$, therefore the limit $\lim\limits_{x\to 3} \dfrac{2x-6}{|x-3|}$ doesn't exist

4. Compute $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{\sin x}$

   Solution: $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{\sin x} = \lim\limits_{x\to 0} \dfrac{\cos x - 1}{\sin x} \times \dfrac{\cos x + 1}{\cos x + 1} = \lim\limits_{x\to 0} \dfrac{\cos^2 x - 1}{\sin x(\cos x + 1)} =$
   $\lim\limits_{x\to 0} \dfrac{-\sin^2 x}{\sin x(\cos x+1)} = \lim\limits_{x\to 0} \dfrac{-\sin x}{\cos x + 1} = \dfrac{\lim\limits_{x\to 0}(-\sin x)}{\lim\limits_{x\to 0}(\cos x + 1)} = \dfrac{-\sin 0}{\cos 0 + 1} = 0$

5. Compute $\lim\limits_{x\to 0} x^5 \cos \dfrac{3}{x}$

   Solution: Since $|\cos\frac{3}{x}| \leq 1$, $0 \leq |x^5 \cos\frac{3}{x}| = |x^5||\cos\frac{3}{x}| \leq |x^5|$, so $x^5\cos\frac{3}{x}$ is bounded between $-x^5$ and x^5. Actually, when $x > 0$, $-x^5 \leq x\cos\frac{3}{x} \leq x^5$ and when $x < 0$, $x^5 \leq x\cos\frac{3}{x} \leq -x^5$. We know $\lim\limits_{x\to 0} x^5 = \lim\limits_{x\to 0}(-x^5) = 0$, then by the Squeeze Theorem, $\lim\limits_{x\to 0} x^5 \cos\dfrac{3}{x} = 0$.

6. If $f(x) = \begin{cases} x^3+1, x \geq 2 \\ x-1, x < 2 \end{cases}$, compute $\lim_{x\to 2} f(x)$

   Solution:

   $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^3+1 = 2^3+1 = 9$

   $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x-1 = 2-1 = 1$

   We see $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$, so $\lim_{x\to 2} f(x)$ does not exist

7. If $f(x) = \begin{cases} \sin x, x \geq 0 \\ \frac{\sin x}{x} - 1, x < 0 \end{cases}$, compute $\lim_{x\to 0} f(x)$

   Solution:

   $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sin x = \sin 0 = 0$

   $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (\frac{\sin x}{x} - 1) = (\lim_{x\to 0^-} \frac{\sin x}{x}) - 1 = 1 - 1 = 0$

   We see $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 0$, so $\lim_{x\to 0} f(x) = 0$

8. If $f(x) = \begin{cases} x^2, x \neq 0 \\ 1, x = 0 \end{cases}$, compute $\lim_{x\to 0} f(x)$

   Solution: $\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 = 0^2 = 0$