1. Express the limit as a definite integral on the interval $[2, 6]$: $\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{\cos x_i^*}{x_i^*} \Delta x_i$

   Solution: $\int_2^6 \frac{\cos x}{x}\, dx$

2. Use the Midpoint Rule with $n = 5$ to estimate the integral $\int_0^2 \frac{x}{x+1}\, dx$. Round the answer to four decimal places.

   Solution:

$$\begin{aligned}\int_0^2 \frac{x}{x+1}\, dx &\approx \left(\frac{0.2}{0.2+1} + \frac{0.6}{0.6+1} + \frac{1}{1+1} + \frac{1.4}{1.4+1} + \frac{1.8}{1.8+1}\right) \times 0.4 \\ &= 0.9071\end{aligned}$$

3. Evaluate the integrals:

   (i). $\int_{-2}^{3} (x^2 - 3)\, dx$

   Solution:

   $\int_{-2}^{3} (x^2 - 3)\, dx = \frac{x^3}{3} - 3x\Big|_{-2}^{3} = -\frac{10}{3}$

   (ii). $\int_0^{\frac{\pi}{4}} \frac{1+\cos^2 x}{\cos^2 x}\, dx$

   Solution:

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \frac{1+\cos^2 x}{\cos^2 x}\, dx &= \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x} + 1\, dx \\ &= \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x}\, dx + \int_0^{\frac{\pi}{4}} 1\, dx \\ &= \tan x|_0^{\frac{\pi}{4}} + x|_0^{\frac{\pi}{4}} \\ &= 1 + \frac{\pi}{4}\end{aligned}$$

   (iii). $\int_{-1}^{1} e^{x+1}\, dx$

   Solution:

$$\begin{aligned}\int_{-1}^{1} e^{x+1}\,dx &= \int_{-1}^{1} e^{x} e\,dx\\ &= e\int_{-1}^{1} e^{x}\,dx\\ &= e\; e^{x}\big|_{-1}^{1}\\ &= e(e-\frac{1}{e})\\ &= e^2-1\end{aligned}$$

(iv). $\int_0^{\frac{3\pi}{2}} |\sin x|\,dx$

Solution:

$$\begin{aligned}\int_0^{\frac{3\pi}{2}} |\sin x|\,dx &= \int_0^{\frac{\pi}{2}} |\sin x|\,dx + \int_{\pi}^{\frac{3\pi}{2}} |\sin x|\,dx\\ &= \int_0^{\pi} \sin x\,dx + \int_{\pi}^{\frac{3\pi}{2}} -\sin x\,dx\\ &= -\cos x|_0^{\pi} + \cos x|_{\pi}^{\frac{3\pi}{2}}\\ &= 2+1\\ &= 3\end{aligned}$$

4. Find the general indefinite integral $\int \frac{\sin 2x}{\sin x}\,dx$

   Solution:

$$\begin{aligned}\int \frac{\sin 2x}{\sin x}\,dx &= \int \frac{2\sin x\cos x}{\sin x}\,dx\\ &= \int 2\cos x\,dx\\ &= 2\sin x + C\end{aligned}$$

5. The velocity of a moving particle is $v(t) = t^2 - 2t - 8$.

(i). Find the displacement during the time $[1, 6]$.

Solution:

$$\begin{aligned}\int_1^6 v(t)\,dt &= \int_1^6 t^2 - 2t - 8\,dt \\ &= \frac{t^3}{3} - t^2 - 8t\Big|_1^6 \\ &= -\frac{10}{3}\end{aligned}$$

(ii). Find the distance traveled by the particle during time $[1, 6]$.

Solution:

$v(t) = t^2 - 2t - 8 = (t+2)(t-4)$, so $v(t) > 0$ when $t < -2$ or $t > 4$, $v(t) < 0$ when $-2 < t < 4$.

$$\begin{aligned}\int_1^6 |v(t)|\,dt &= \int_1^4 |v(t)|\,dt + \int_4^6 |v(t)|\,dt \\ &= \int_1^4 -v(t)\,dt + \int_4^6 v(t)\,dt \\ &= \int_1^4 -t^2 + 2t + 8\,dt + \int_4^6 t^2 - 2t - 8\,dt \\ &= \left(-\frac{t^3}{3} + t^2 + 8t\right)\Big|_1^4 + \left(\frac{t^3}{3} - t^2 - 8t\right)\Big|_4^6 \\ &= \frac{98}{3}\end{aligned}$$