1. Find the most general antiderivative of $f(x) = x\sqrt{x} - \frac{2}{x^2+1} + \sin x$.

   Solution:

   $f(x) = x^{\frac{3}{2}} - \frac{2}{x^2+1} + \sin x$, so the most general form of antiderivative is

   $F(x) = \frac{2}{5}x^{\frac{5}{2}} - 2\tan^{-1} x - \cos x + C$

2. Find $f(x)$ if $f''(x) = -2 + 12x - 12x^2$ and $f(0) = 4$, $f'(0) = 12$.

   Solution:

   $f'(x) = -2x + 6x^2 - 4x^3 + C_1$. $12 = f'(0) = C_1$, so $f'(x) = -2x + 6x^2 - 4x^3 + 12$.

   $f(x) = -x^2 + 2x^3 - x^4 + 12x + C_2$, $4 = f(0) = C_2$,

   so $f(x) = -x^2 + 2x^3 - x^4 + 12x + 4$

3. A particle is moving with acceleration $a(t) = 10\sin t + 3\cos t$, $s(0) = 0$, $s(1) = 20$. Find the position function $s(t)$.

   Solution:

   $v(t) = -10\cos t + 3\sin t + C_1$, $s(t) = -10\sin t - 3\cos t + C_1 t + C_2$.

   $0 = s(0) = -3 + C_2$, so $C_2 = 3$, $s(t) = -10\sin t - 3\cos t + C_1 t + 3$.

   $20 = s(1) = -10\sin 1 - 3\cos 1 + C_1 + 3$, so $C_1 = 17 + 10\sin 1 + 3\sin 1$, we get:

   $s(t) = -10\sin t - 3\cos t + (17 + 10\sin 1 + 3\cos 1)t + 3$

4. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.

   Solution:

   $f(x) = \frac{x^4}{4} + C$. Assume $y = f(x)$ is tangent to $x + y = 0$ at (x_0, y_0). Since (x_0, y_0) is on $x + y = 0$, $x_0 + y_0 = 0$, so the point is $(x_0, -x_0)$.

   The slope of f at $x = x_0$ equals to $x + y = 0$, so $f'(x_0) = x_0^3 = -1$, we get $x_0 = -1$, so $(x_0, -x_0) = (-1, 1)$ is on $y = f(x)$, i.e. $1 = \frac{(-1)^4}{4} + C$, we get $C = \frac{3}{4}$, so $f(x) = \frac{x^4}{4} + \frac{3}{4}$

5. Use R_n to compute the area under $y = 2x + 1$ and between $x = 0$ and $x = 1$.

   Solution: $\Delta x = \frac{1}{n}$, $x_i = \frac{i}{n}$, so

$$
\begin{aligned}
R_n &= \sum_{i=1}^{n}(2x_i+1)\Delta x \\
&= \sum_{i=1}^{n}(\frac{2i}{n}+1)\frac{1}{n} \\
&= \sum_{i=1}^{n}\frac{2i}{n^2}+\sum_{i=1}^{n}\frac{1}{n} \\
&= \sum_{i=1}^{n}\frac{2}{n^2}\sum_{i=1}^{n} i+\frac{1}{n}\sum_{i=1}^{n}1 \\
&= \frac{2}{n^2}\frac{n(n+1)}{2}+\frac{1}{n}\times n \\
&= \frac{n-1}{n}+1
\end{aligned}
$$

So the area is $\lim_{n\to\infty} R_n = \lim_{n\to\infty}\frac{n-1}{n}+1=2$

6. Compute L_4 for $y=\sin x$ on $[0,\pi]$.

   Solution: $\Delta x=\frac{\pi}{4}$. $x_0=0, x_1=\frac{\pi}{4}, x_2=\frac{\pi}{2}, x_3=\frac{3\pi}{4}, x_4=\pi$.

$$
\begin{aligned}
L_4 &= \sum_{i=1}^{4}\sin x_{i-1}\Delta x \\
&= (\sin 0+\sin\frac{\pi}{4}+\sin\frac{\pi}{2}+\sin\frac{3\pi}{4})\frac{\pi}{4} \\
&= (0+\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2})\frac{\pi}{4} \\
&= (\sqrt{2}+1)\frac{\pi}{4}
\end{aligned}
$$