1. $p > 3$ is a prime and $p \equiv 2 \pmod{3}$. $G$ is a group of order $3p$. Prove $G \cong \mathbb{Z}/3p\mathbb{Z}$.

2. Prove $Aut(S_3) \cong S_3$.

3. Prove the unit quaternion group $Q_8$ is not a semidirect product of its proper subgroups.


5. A **short exact sequence** of groups is a sequence of groups and homomorphisms:

\[
\{1\} \longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow \{1\}
\]

such that the image of each map equals to the kernel of the next map. For example, $\text{Im}(f) = \ker(g)$. Given the above short exact sequence, prove:

(i). $f : A \longrightarrow B$ is injective  
(ii). $g : B \longrightarrow C$ is surjective  
(iii). $B / f(A) \cong C$  
(iv). Given two groups $G$ and $G'$ and a homomorphism $\phi : G' \longrightarrow Aut(G)$, prove the following is a short exact sequence:

\[
\{1\} \longrightarrow G \xrightarrow{i_1} G \rtimes_\phi G' \xrightarrow{\pi_2} G' \longrightarrow \{1\}
\]

where $i_1(g) = (g, 1')$ and $\pi_2(g, g') = g'$ for any $g \in G, g' \in G'$. 