1. Prove any group of order 77 is cyclic.

2. Prove a group of order 90 is not simple.

3. \( G \) is a group with \(|G| = pq\), where \( p, q \) are primes. Prove \( G \) is not simple.

4. \( G \) is a simple group of order 168. How many elements of order 7 are there in \( G \)?

5. \( \phi : \mathbb{Z}/2\mathbb{Z} \rightarrow Aut(\mathbb{Z}) \) is defined by
   \[
   \phi(\bar{m}) : \mathbb{Z} \rightarrow \mathbb{Z} \\
   k \mapsto (-1)^m k
   \]
   Let \( G = \mathbb{Z} \rtimes_{\phi} (\mathbb{Z}/2\mathbb{Z}) \). Find all the elements of finite order in \( G \).

6. Prove \( O_2(\mathbb{R}) = SO_2(\mathbb{R}) \rtimes < r > \), where \( r = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \)

7. \( G \) and \( G' \) are groups. \( \phi : G' \rightarrow Aut(G) \) is a homomorphism. Prove \( G \rtimes_{\phi} G' \) is abelian if and only if \( G, G' \) are both abelian and \( \phi \) is the trivial homomorphism.