1. A is a real $n \times n$ matrix. Prove $A \in O_n(\mathbb{R})$ if and only if the rows (columns) of $A$ are unit vectors that are pairwisely perpendicular.

2. Prove that a linear operator on $\mathbb{R}^2$ is a reflection if and only if its eigenvalues are 1 and $-1$, and the eigenvectors with these eigenvalues are orthogonal.

3. Prove that every matrix in $SO_3$ has an eigenvalue $\lambda = 1$. Is it still true for $O_3$?

4. Let $H = \{t_\tilde{a} \rho_\theta \in M_2|\tilde{a} \in \mathbb{Z} \times \mathbb{Z}, \theta = \frac{\pi k}{2}, k \in \mathbb{Z}\}$. Prove $H$ is a subgroup of $M_2$, where $M_2$ is the group of isometries on the plane.

5. Prove that $\rho_\theta r^k = \rho_\omega r^l$ in $O_2$ if and only if $\theta - \omega = 2\pi m$ for some $m \in \mathbb{Z}$ and $\bar{k} \equiv \bar{l} \pmod{2}$

6. Define a map
   \[ \Psi : M_2 \longrightarrow \{\pm 1\} \]
   \[ t_\tilde{a} \rho_\theta r^k \mapsto (-1)^k \]
   Prove $\Psi$ is a well-defined homomorphism.

7. The plane $\mathbb{R}^2$ has another description, i.e. the complex plane $\mathbb{C}$. The identification is $(x, y) \in \mathbb{R}^2$ corresponds to $z = x + iy \in \mathbb{C}$. Under this identification, describe each of the following concepts in terms of complex numbers:
   (i). Inner Product of two plane vectors
   (ii). Distance between two points
   (iii). Translations
   (iv). Rotation centered at origin with angle $\theta$
   (v). Reflection along $x$-axis
   (vi). $r \rho_\theta = \rho_{-\theta} r$