1. Write \((1\ 3\ 7)(2\ 4\ 5\ 6)\in S_7\) as a product of 2-cycles.

2. Compute the signature of the element \((1\ 2\ 3)(6\ 7)(4\ 5\ 9)\in S_9\).

3. \(H\) is a subgroup of \(S_n\) with \(|H|\) an odd number. Prove \(H \subseteq A_n\).
   
   (Hint: Consider \([H : H \cap A_n]\))

4. \(\tau, \sigma \in S_n\). Prove that \(\tau\sigma\) and \(\sigma\tau\) have the same cycle type.

5. If \(H\) is a normal subgroup of \(G\) and \(K\) is a normal subgroup of \(H\), is it always true that \(K\) is a normal subgroup of \(G\)? If yes, prove it; if no, provide a counter-example.

6. If \(n \geq 5\), prove the only proper normal subgroup of \(S_n\) is \(A_n\).