1. $H$ and $K$ are subgroups of $G$, $x, y \in G$. Prove if $xH \cap yK \neq \emptyset$, then $xH \cap yK = g(H \cap K)$ for some $g \in G$.

2. If $G$ is a group of order $p^n$, where $p$ is a prime and $n > 1$. Prove $G$ contains an element of order $p$.

3. If $G$ has five subgroups of order 7, prove $G$ has at least 35 elements.

4. $G$ is a group of order 25. If $G$ has only one subgroup of order 5, prove $G$ is cyclic.

5. Prove that every subgroup of index two is a normal subgroup.

6. $\bar{a} \in \mathbb{Z}/n\mathbb{Z}$ and $\bar{a} \neq \bar{0}$, what is the order of $\bar{a}$ in $\mathbb{Z}/n\mathbb{Z}$?

7. Is $\text{Aut}(\mathbb{Z}/8\mathbb{Z})$ isomorphic to $\text{Aut}(\mathbb{Z}/10\mathbb{Z})$? Why?

8. $m \geq 2, n \geq 2$ are positive integers and they are relatively prime. $a, b \in \mathbb{Z}$. Prove there exists $k, l \in \mathbb{Z}$ such that $x = anl + bmk$ is a solution to the system of equations

\[
\begin{cases}
x \equiv a \pmod{m} \\
x \equiv b \pmod{n}
\end{cases}
\]