1. Let $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ be given by $f(m, n) = 2m + n$. Discuss whether $f$ is injective, surjective, bijective.

2. $\phi : X \rightarrow Y$ is a function from a set $X$ to a set $Y$. $Z$ is a set. Define $M(Z, X)$ to be the set of all functions from $Z$ to $X$, and define $M(Z, Y)$ to be the set of all functions from $Z$ to $Y$. Define

$$\Phi : M(Z, X) \rightarrow M(Z, Y)$$

$$f \mapsto \phi \circ f$$

If $\phi$ is injective, prove $\Phi$ is injective.

3. $G$ is a group, $x, y \in G$. Show by induction that $(yxy^{-1})^n = yx^ny^{-1}$ for any positive integer $n$.

4. Let $G$ be the set of all functions $\mathbb{R} \rightarrow \mathbb{R}$. Given $f_1$ and $f_2$ in $G$, define $f_1 + f_2$ to be the function $(f_1 + f_2)(x) = f_1(x) + f_2(x)$ for any $x \in \mathbb{R}$. Show that $G$ is an abelian group with the above law of composition.

5. Prove the set of all $n \times n$ matrices with real entries, $M_n(\mathbb{R})$, is an abelian group if we define the law of composition to be addition of matrices.

6. $G$ is a group.
   (i). If $g \in G$, prove the inverse element of $g$ is unique in $G$.
   (ii). If $a, b \in G$, prove $(ab)^{-1} = b^{-1}a^{-1}$
   (iii). If $x, y, z \in G$ and $xyz = 1$, prove $yzx = 1$

7. $G = \{1, -1\}$ is the group with composition to be multiplication of numbers. Draw the multiplication table for $G$. 
