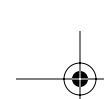


Robust Portfolio Optimization and Management





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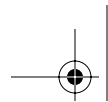
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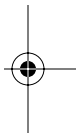
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FRANK J. FABOZZI

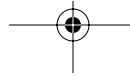
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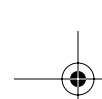
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SERGIO M. FOCARDI



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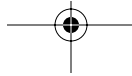
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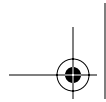
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FJF

To my wife Donna and my children,
Francesco, Patricia, and Karly

PNK

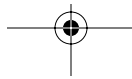
To Åke and Gunilla, my parents, and to John and Carmen,
my wife's parents, for their unending love and support

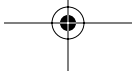
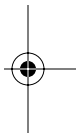
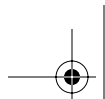
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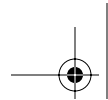
To my husband, Christian Hicks,
and in memory of my grandfather, Georgyi Milyankov

SMF

To the memory of Bertrand Russell to whom I owe
the foundation of my intellectual development

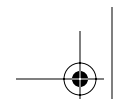






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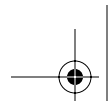
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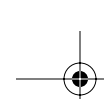
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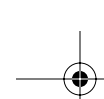
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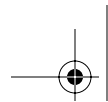
Preface

In the past few years, there has been a notable increase in the use of financial modeling and optimization tools in equity portfolio management. In addition to the pressure on asset management firms to reduce costs and maintain a more stable and predictable performance in the aftermath of the downturn in the U.S. equity markets in 2002, three other general trends have contributed to this increase. First, there has been a revived interest in predictive models for asset returns. Predictive models assume that it is possible to make conditional forecasts of future returns—an objective that was previously considered not achievable by classical financial theory. Second, the wide availability of sophisticated and specialized software packages has enabled generating and exploiting these forecasts in portfolio management, often in combination with optimization and simulation techniques. Third, the continuous increase in computer speed and the simultaneous decrease in hardware costs have made the necessary computing power affordable even to small firms.

As the use of modeling techniques has become widespread among portfolio managers, however, the issue of how much confidence practitioners can have in theoretical models and data has grown in importance. Consequently, there is an increased level of interest in the subject of robust estimation and optimization in modern portfolio management. For years, robustness has been a crucial ingredient in the engineering, statistics, and operations research fields. Today, these fields provide a rich source of ideas to finance professionals. While robust portfolio management undoubtedly demands much more than the robust application of quantitative techniques, there is now a widespread recognition for the need of a disciplined approach to the analysis and management of investments.

In this book we bring together concepts from finance, economic theory, robust statistics, econometrics, and robust optimization, and illustrate that they are part of the same theoretical and practical environment—in a way that even a nonspecialized audience can understand and appreciate. At the same time, we emphasize a practical treatment of the subject, and translate complex concepts into real-world applications for robust return





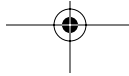
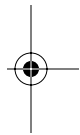
forecasting and asset allocation optimization. Thereby, we address a number of issues in portfolio allocation and rebalancing. In particular, we discuss how to make portfolio management robust with respect to model risk, long-term views of the market, and market frictions such as trading costs.

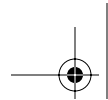
The book is divided into four parts. Part I covers classical portfolio theory and its modern extensions. We provide an up-to-date treatment of methods for advanced risk management, nonnormal distributions for asset returns, transaction costs, and multiaccount portfolio management. Part II introduces traditional and modern frameworks for robust estimation of returns. We address a number of topics that include dimensionality reduction, robust covariance matrix estimation, shrinkage estimators, and the Black-Litterman framework for incorporating investors' views in an equilibrium framework. Part III provides readers with the necessary background for handling the optimization part of portfolio management. It covers major issues in numerical optimization, introduces widely used optimization software packages and modeling platforms, and discusses methods for handling uncertainty in optimization models such as stochastic programming, dynamic programming, and robust optimization. Part IV focuses on applications of the robust estimation and optimization methods described in the previous parts, and outlines recent trends and new directions in robust portfolio management and in the investment management industry in general. We cover a range of topics from portfolio resampling, robust formulations of the classical portfolio optimization framework under modeling uncertainty, robust use of factor models, and multiperiod portfolio allocation models—to the use of derivatives in portfolio management, currency management, benchmark selection, modern quantitative trading strategies, model risk mitigation, as well as optimal execution and algorithmic trading.

We believe that practitioners and analysts who have to develop and use portfolio management applications will find these themes—along with the numerous examples of applications and sample computer code—useful. At the same time, we address the topics in this book in a theoretically rigorous way, and provide references to the original works, so the book should be of interest to academics, students, and researchers who need an updated and integrated view of the theory and practice of portfolio management.

TEACHING USING THIS BOOK

This book can be used in teaching courses in advanced econometrics, financial engineering, quantitative investments and portfolio manage-





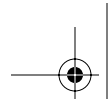
ment, as the main course book, as supplemental reading on advanced topics, and/or for student projects. The material in Chapters 2 through 11 of the book is appropriate for undergraduate advanced electives on investment management, and all topics in the book are accessible to graduate students in finance, economics or in the mathematical and physical sciences. The material is also appropriate for use in advanced graduate electives in the decision sciences and operations research that focus on applications of quantitative techniques in finance.

For a typical course, it is natural to start with Chapters 2, 5, and 6 where modern portfolio and asset pricing theory and standard estimation techniques are covered. Basic practical considerations are presented in Chapters 4 and 11. Chapters 3, 7, 8, 10, 12, and 13 are more advanced and do not have to be covered in full. A possibility is to focus on the most common techniques used in portfolio management today, such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) (in Chapter 3), shrinkage estimators and the Black-Litterman model (in Chapter 8), robust optimization (in Chapters 10 and 12), and transaction costs and portfolio rebalancing (in Chapter 13). Student projects can be based on specialized topics such as multiaccount optimization (in Chapter 4), numerical optimization techniques (in Chapter 9), modern trading strategies, optimal execution, and algorithmic trading (in Chapter 14).

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Frank J. Fabozzi
Petter N. Kolm
Dessislava A. Pachamanova
Sergio M. Focardi



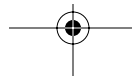


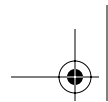
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