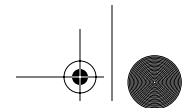


Financial Modeling of the Equity Market





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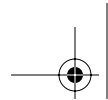
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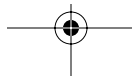
From CAPM to Cointegration

FRANK J. FABOZZI
SERGIO M. FOCARDI
PETTER N. KOLM



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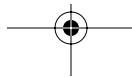
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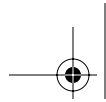
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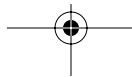
*To my wife Donna and my children,
Francesco, Patricia, and Karly*

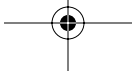
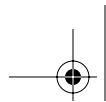
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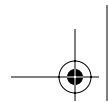
*To the memory of Bertrand Russell to whom I owe
the foundation of my intellectual development*

PNK

To my best friend, my wife, and my love—Carmen







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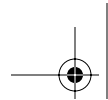
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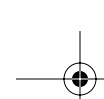
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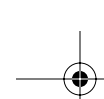
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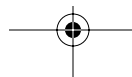
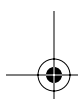
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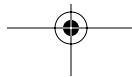
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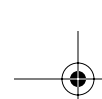
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Preface

This book is about financial modeling for equity asset management. We take a broad view of financial modeling, encompassing pure modeling as well as model engineering and financial optimization. Our perspective is that of an asset management firm. When reasoning and making decisions about modeling, a firm needs to grasp all the aspects related to modeling. This includes not only the mathematical models per se but also methods for model estimation, the optimization process that translates model forecasts into active strategies, and methods that help mitigate eventual inadequacies of the models being used.

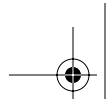
Our perspective is similar to that of physical engineering, where the knowledge of a few abstract laws of physics is a far cry from building an automobile or an airplane. We broadly define financial modeling as theoretical financial and mathematical principles as well as statistical methods that allow for representing and forecasting financial data, procedures for estimating and testing these representations, and methods for engineering and optimizing financial strategies. Without a methodology for engineering, estimating, and testing financial strategies, a financial model is of little use.

In this book we offer an up-to-date treatment of financial modeling for asset management, presenting and discussing a number of developments at the forefront of equity modeling technology: robust estimation, robust optimization, the analysis of transaction costs, linear and non-linear dynamic models, and model risk mitigation techniques.

Since the downturn in the U.S. equity market in 2002, there has been an increased use of financial modeling and optimization in equity portfolio management. Under pressure to boost returns and reduce costs, asset management firms have begun to look with increasing attention at quantitative techniques. Not only has the diffusion of quantitative methods in equity portfolio management broadened since the turn of the century, but the variety of models and depth of use have also increased.

Three trends are worth pointing out. First, there is a greater use of predictive models. Predictive models assume that it is possible to make conditional forecasts of expected returns, an objective that was previously considered not achievable by classical financial theory. Second, in





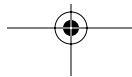
order to exploit forecasts, optimization techniques are now being used. Previously, optimization technologies were considered too brittle for safe deployment in asset management. Third, as a consequence of a greater use of predictive models and optimization, there is a growing interest in “robust” methods—particularly methods for robust estimation and robust optimization—as well as a heightened attention to the analysis of transaction costs.

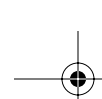
Two technology trends have also facilitated the deployment of modeling in equity asset management. First, the continuous decrease in the cost of computers coupled with a parallel increase in computational power makes the necessary computing power affordable even to small firms. Second, statistical software packages now offer a broad variety of general and specialized econometric building blocks. The availability of these software packages proved to be a powerful enabler for the deployment of modeling.

The book is divided into four parts. In Part One we cover modern portfolio theory, numerical optimization methods, the analysis of transaction costs, and the handling of nonnormal distributions in portfolio allocation applications through the consideration of higher moments. We present important recent theoretical advances as well as the basic modeling techniques. In Part One these methods are applied in the classical one-period mean-variance and utility-maximization frameworks. This allows us to give an up-to-date treatment of modern portfolio theory and to explain new methods of analysis of transaction costs, numerical optimization, and the handling of higher moments in a unified and concrete framework.

In Part Two we introduce robust methodologies. As mentioned above, robust techniques have become fundamental in the practical deployment of modern portfolio theory. We discuss both the classical and more recent methods for forecasting expected return and risk. In particular, we address topics including dimensionality reduction and the robust estimation of the covariance matrix of returns. Part Two provides a comprehensive presentation of robust methodologies for estimation and optimization.

In Part Three we discuss the motivation for adopting predictive models and present several families of models. We begin with an analysis of the empirical evidence of feedbacks in financial markets. We then describe the statistical properties of models that allow to capture these feedbacks, including regressive and autoregressive models, state-space models, and nonlinear hidden variable, regime-switching models. We discuss cointegration and its many different representations, including dynamic factor analysis. We also elaborate on the process and the pitfalls of the model selection process.



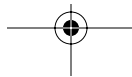


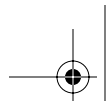
In Part Four we discuss current methods for estimating dynamic models. We close with a discussion on how to mitigate model risk in a dynamic framework.

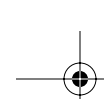
Two appendices provide complementary mathematical details for the interested reader. Appendix A discusses solutions of difference equations. Appendix B presents a number of mathematical facts on regressions, correlations, and copulas. In several chapters throughout the book we make use of the MSCI World Index and its individual constituents (country indices) in various illustrations. Appendix C provides some basic statistics and properties of this data set.

The purpose of this book is to serve as a working tool for practitioners who use financial modeling in their work and for students who are pursuing careers in finance. Since most of the subjects are advanced in nature, we have tried to offer an intuitive and simplified treatment of most mathematical topics, although at no time have we compromised mathematical rigor. When we feel the subject is too technical, we offer references to the original work. In summary, we feel the book should be of interest to practitioners, students, and researchers who need an updated and integrated view of equity modeling.

Frank J. Fabozzi
Sergio M. Focardi
Petter N. Kolm





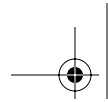


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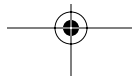


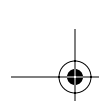
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