A convex optimization approach to generalized moment problems in multivariate spectral estimation

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Multidimensional spectral estimation is an old and challenging problem which keeps generating widespread interest in the natural and engineering sciences. In this talk, we present effective multivariate generalizations of the THREE-approach introduced by Byrnes, Georgiou and Lindquist in [1, 4]. The latter may be viewed as a (considerable) extension of the Burg-Jaynes Maximum Entropy Method. In the THREE-like estimation methods, the problem is first recast as a generalized moment problem with complexity constraints. Spectral estimation then turns into a constrained spectrum approximation problem where the choice of the (pseudo) distance between spectra is crucial. THREE-like approaches all permit higher resolution in desired frequency bands and are extremely effective in the case of a short observation record.

In [3, 5], an approach based on a generalized *Hellinger distance* motivated by spectral factorization concepts was developed. In [2], a multivariate version of the *Itakura-Saito distance* was employed to solve the state-covariance matching problem. The choice of this *Bregman divergence* was there motivated by a profound, far reaching connection between relative entropy in the time and spectral domains. The corresponding solution entails a *complexity upper bound* which improves on the one so far available in the multichannel framework. Indeed, it is *equal* to the one featured by THREE in the scalar case. The solution is computed via a globally convergent matricial Newton-type algorithm. Simulation suggests the effectiveness of the new technique especially in the case of short data records where it outperforms MATLAB's PEM and MATLAB's N4SID.

References

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