

Multireference alignment, bispectrum inversion and cryo-EM

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In this talk, we consider the problem of estimating a signal from multiple noisy translated versions of itself, called *multireference alignment*. Most existing methods rely on estimating the relative translations, which is impossible below a critical signal-to-noise ratio. To overcome this fundamental barrier, we propose estimating the signal directly using translation-invariant features. Specifically, we estimate the signal's Fourier magnitudes and phases from, respectively, the mean power spectrum and bispectrum of the observations. To estimate the Fourier phases from the bispectrum, we propose and analyze several algorithms. Our main method consists of a non-convex local optimization algorithm over the smooth manifold of phases. Empirically, in the absence of noise, the non-convex algorithm consistently converges to the target signal regardless of initialization. The algorithm also appears to be robust to noise. We propose additional algorithms based on alternating phase synchronization, convex relaxation, frequency marching and integer programming. In the last part of the talk, we will discuss how similar ideas can be applied to cryo-electron microscopy.

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