

Life and Cognition Through Keyhole of Mathematics, I. What is Life? Fall 2024

In the process of rewriting and editing

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October 6, 2024

WHAT BRINGS A MATHEMATICIAN TO THINKING ABOUT LIFE?
WHAT IS LIFE? WHAT IS COGNITION? WHAT IS "MATHEMATICS"?

- Human (and animal) perception systems, e.g. vision, are especially apt at recognition whichever associated with Life;¹ mathematician's mind is excited by the structural harmony of the images offered by Life.
- Life is the structure of live entities, the full mathematical beauty² of which, unravels on the sub-cellular and molecular levels, where, unlike bewildering diversity the manifestations of life around us, Life is rooted in a few basic principles.
- We want to identify general principles, which underline common structural features of Life, Cognition, Language and Mathematics.
- Deeper understanding of the nature of Mathematics needs understanding the live structures.
- Pondering over the nature of Life may bring new mathematical ideas.³
- Cooperating with biologists, you may contribute to a solution of a biological problem.⁴
- By finding a logical/mathematical incompleteness/inconsistency in a biologists' description/model, you direct the research toward something new.⁵
- Upon learning more on the stability/instability problem of ecological systems, you may switch from mathematics to developing survival strategies for

¹It takes 200-300 milliseconds for a monkey to recognize an animal image on the computer screen. By comparison, it takes you more than 500 milliseconds to start braking on red (but only 8 milliseconds to decide if an object is suitable for food if you are a star-nosed mole.

²"Beauty" is a psychological, hence, biological, phenomenon – branch of psychology which grows from the ideas of Douglas Spalding. "Mathematics" is the product of the human brain – a small live entity. "Mathematical beauty" is whichever may entertain a mathematician's brain.

³*Von Neumann universal constructor* is an instance of this.

⁴Examples: 1908 *Hardy-Weinberg 1908 rendition of Mendel's equilibrium principle which led to Bernstein algebras*, 1924),

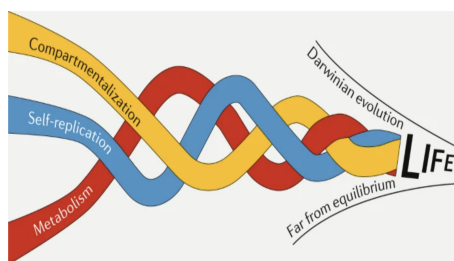
Filler's *supercoiled DNA writhing number*,
mathematical algorithms for *multiple sequence alignments*

⁵Albeit tangentially, mathematicians' concerns on unlinking strands of DNA contributed to the discovery of *topoisomerases*, see W. F Pohl, G W Roberts. *Topological considerations in the theory of replication of DNA* and <https://www.jstor.org/stable/777807?seq=3>

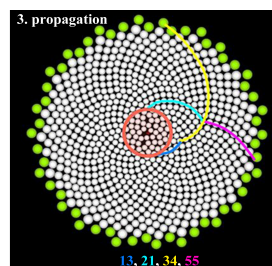
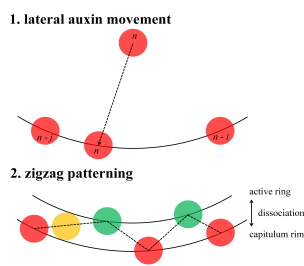
Life on Earth.

- Participating in developing new bio-technologies, you may influence the direction of human civilisation and future progress of mathematics itself.
- There is much to learn from biology; learning new things keeps your brain active.

ADD! directionality of transcription
 bio-information $BIO \leq SH$ Shannon information but may lead to a source of »SH
 specificity/universality of mechanisms of cell signalling
 universality of posttranslational modifications, e.g. phosphorylation
 emphasis on function rather than on physical/chemical nature, e.g. in phosphorylation and methylation, acetylation
 2 ways making units : close genetic similarity and spacial /functional connections
 hierarchy of regulation processes that are energy, matter and information flows, including regulation processes themselves
 energy consumption on different levels
 multiple levels of description of live systems, from molecules to populations with non-physical (discrete discontinuous) relations between them: a change in genome may affect a population
 The large scale difference of viruses/bacteria concentrations and the same involving small molecules, instead of viruses
 Models/examples: structures of basic cells, ant hills, evolution of ???
 basic features of information different from physical systems are types of transformation e.g. compression of information, levels of universality and ????
 Logical loops of the egg↔chicken kind
 Can (imaginary) life reduced to a single organism (Ocean of Stanislaw Lem) be stable.
 What are other imaginary "life sysetems" obtained by modifying basic **features of life on Earth seen on different levels**



Life



Math

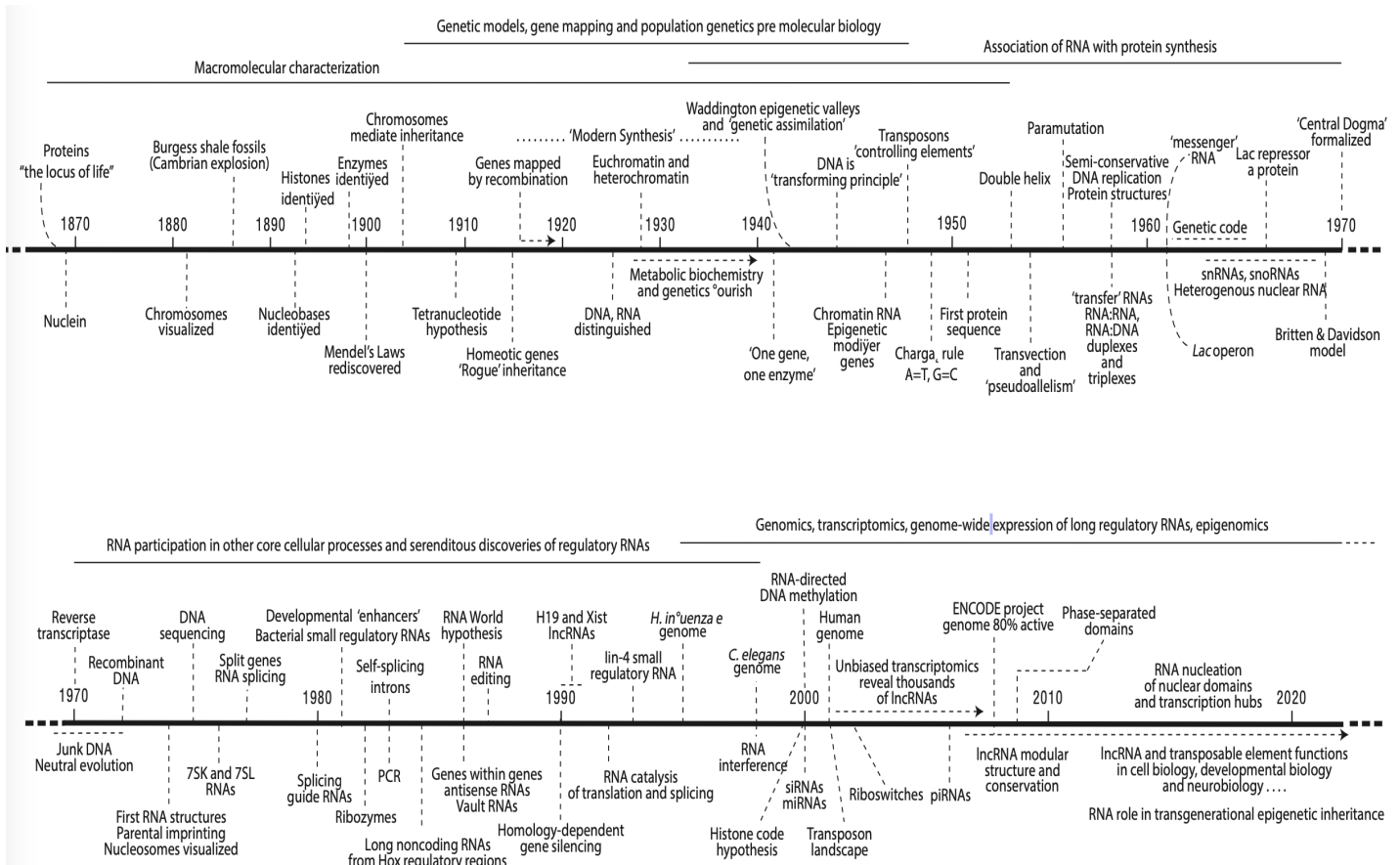
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...combinaisons, d'arrangements, de causes, d'effets, de principes, qui tous concourent au même but, et que nous ne connaissons que par des résultats si difficiles à comprendre. Georges-Louis Buffon, Histoire générale des animaux, 1749.⁷

⁶<https://www.nature.com/articles/s41570-020-0196-x>

⁷https://www.azquotes.com/author/19916-Georges_Louis_Leclerc_Comte_de_Buffon

Contents



GENETIC INFORMATION JOHN MATTICK & PAULO AMARAL

BOOKS:

Protein Physics 2nd Edition A Course of Lectures by Alexei Finkelstein Oleg Ptitsyn.

The Science and Technology Behind the Human Genome Project, by Charles R. Cantor and Cassandra L.

Unraveling DNA: The Most Important Molecule Of Life, Revised And Updated Edition by Frank-Kamenetskii.

The Logic of Chance: The Nature and Origin of Biological Evolution by Eugene V. Koonin.

⁸One needs Buffon's level of understanding physics (after Newton), chemistry (after Lavoisier) and mathematics (after Bernoulli and Euler) to say I know that I know nothing about Life.

The Plausibility of Life Resolving Darwin's Dilemma by Marc W. Kirschner and John C. Gerhart.

Cell Biology by the Numbers by Ron Milo, Rob Phillips.

Lateral DNA Transfer: Mechanisms and Consequences by Frederic Bushman.

Landmark Experiments in Molecular Biology Michael Fry.

RNA, the Epicenter of Genetic Information A new understanding of molecular biology by John Mattick and Paulo Amaral.

1 Universality/Specificity of Living Structures&Processes and Bio-mathematical Language of Life

CHESTERTON 1925.

*A dead thing can go with the stream, but only a living thing can go against it. A dead dog can be lifted on the leaping water with all the swiftness of a leaping hound; but only a live dog can swim backwards.*⁹

SCHRÖDINGER 1944.

...[an organism] can only keep ... alive, by continually drawing from its environment negative entropy– which is something very positive... .

*the most essential part of a living cell - the chromosome fibre may suitably be called an aperiodic crystal.*¹⁰

PROSS 2012.

*Life is a selfsustaining kinetically stable dynamic reaction network derived from the replication reaction. Biology is just an elaborate extension of replicative chemistry ...*¹¹

HORDIJK&STEEL 2018

*...an autocatalytic (replication) network, where a set of molecules catalyses an identical second set.*¹²

Organisms are *particular* thermodynamic systems but this "particularity" can't be expressed in the language of physics:

this language is adapted to expressing properties of generic systems and those with logically simple specificity, e.g. symmetry.

Chemistry comes a couple of steps closer to "the essence of Life"; yet Life is not "just elaborated" chemistry: there is no words in a chemist' dictionary to say what kind of elaboration this is.

⁹<https://www.worldinvisible.com/library/chesterton/everlasting/part2c6.htm>
"Dead" motorboats do the same, but these are made by living things anyway.

¹⁰What Is Life? The Physical Aspect of the Living Cell.

<https://www.arvindguptatoys.com/arvindgupta/whatislife-schrodinger.pdf>

https://en.wikipedia.org/wiki/What_Is_Life%3F

¹¹What is Life?: How Chemistry Becomes Biology (Oxford Landmark Science) Illustrated Edition, Kindle Edition by Addy Pross

¹²Autocatalytic Networks at the Basis of Life's Origin and Organization by Wim Hordijk and Mike Steel, Life (Basel). 2018 Dec; 8(4): 62. Published online 2018 Dec 8. doi: 10.3390/life8040062 PMID: PMC6315399 PMID: 30544834.

A mathematician may get a rough idea of the difficulty in defining Life by thinking how to (approximately) describe *viable* (say bacterial) genomes *glive* within the space of all five million long four letters sequences.

The subset

$$”genomes_{live}” = \{g_{life}\} \subset \{...\}^{5 \cdot 10^6}$$

makes a small part of $\{...\}^{5 \cdot 10^6}$, but all knowledge accumulated by *biologists* to day¹³ doesn't allow one to say "how small" this small is and even less so to adequately describe/characterise "genomes_{live}".

(The above mathematical set theoretic language seems poorly adapted to this purpose.)

To speak about Life following biologists, the physicist's&chemist's [space-time-matter-energy]-language must be augmented with the idea of *biological information*.

WHAT IS INFORMATION? This is supposed to express the idea of "useful information", where a formal mathematical realisation of this idea, if possible at all, can't be universal(absolute) but rather *relative* depending on a particular class of *objects sensitive* to (a specific expression of) this "information".

For instance, the information carried by a sentence spoken in English depends on the acoustic sensitivity of one's ear and on one's knowledge of English.

There are two specific form of information (not only) in biology:

program and *signal*

and also several informal, mathematical/biological concepts related to "information":

biological structure,

biological function (performed by a particular structure),

biological purpose (of a function),

where the meaning of these and similar words are *determined by their use* in sentences written in the "bio-mathematical" language, which, in particular, should allow one to speak of

- "information/program *encoded* and *stored* by a material structure."
- "information/signals *transmitted* by a *matter/energy flow*",
- "information/program, which controls/channels a matter/energy flow"
- "biological structure build by (*a network of*) matter/energy flows."

In what follows we illustrate a use of this kind of bio-math language by a few biological examples (on the introductory textbooks level), where, unlike how it is in math, "true" means "80-90% true".

Higher Level Information Structures and Processes : *Embryogenesis, Muscles-Brain Signalling, Minds and Languages*. Our bio-math language may be suitable for speaking about Life on the cellular and sub-cellular levels

But the logic of the material implementation of the *embryogenesis programs of multicellular organisms* and of the *animal muscle-brain signalling processes*

¹³The number of bacteria on Earth is estimated as $5 \cdot 10^{30}$. This makes the number of different genomes less than 10^{20} . But, the number of different *viable* ones must be much greater, $10^{1000000}$, since $(10^{5 \cdot 10^6}$ long) genomes, remains viable under many (more than $10^{1000000}$), random mutations.

are too complicated for that.
 The same applies to
information processing, which makes the animal cognition,
the information structure of the human mind,
the structure the human language.
 (We say a few words about these in section???)

1.1 Units of Life: Organisms, Cells, DNA, RNA, Proteins, Ribosomes...

The Life as we know it (probably, this applies to any chemically based life in the Universe) is composed of well defined *structurally and functionally indivisible individual units*, where each unit is present in (improbably) *many* (nearly) *identical copies*, as, for instance humans on Earth.

The basic *units of Life on Earth*, (the only Life we know of) are *cells* and *organisms*, where organisms are composed of cells and where the principle constituents of cells are *macromolecules* and *macromolecular complexes*.

The basic matter/energy flows are

★ *Chemical metabolism* within cells, especially (but not only) *synthesis of proteins* (which is more than a mere chemistry).

★★ *Assembly of sub-cellular molecular structures* in cells and production of *new copies* of cells.

According to the structures of their cells, all organisms are divided into *prokaryotes* and *eukaryotes*, where the eukaryotic cells are larger and much more *structurally and functionally sophisticated* than the prokaryotes ones.

Most prokaryotes are unicellular and are classified into *bacteria* and *archaea*, while eukaryotes can be both, unicellular and multicellular.

Animals, *Fungi* and *Plants* are multicellular and *protozoa*, some *algae* and some *fungi* (e.g. yeast) are unicellular ones.

The major heteropolymer molecules in the cells are called **DNA**, **RNA** and **proteins (polypeptides)** and the main macromolecular complexes are *ribosomes*.

DNA. *1-Dimensional Keeper of Digitally Stored Information*. A typical bacterial cell, a couple of microns ($1\mu = 10^{-6}\text{m}$) in diameter, contains a *single* distinguished "master-molecule" that is a (**randomly looking**) chain of a *few million* "molecular letters" A,T,C,G "written on" the (**periodic**) *sugar-phosphate backbone*.

DNA – *deoxyribonucleic acid* is a heteropolymer composed of pairs of four *nucleotides*. each, made up of one of the four *bases*

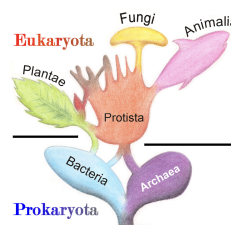
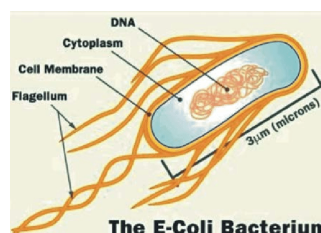
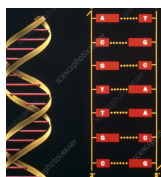


Figure 1: Domains of Life



Schematic
DNA

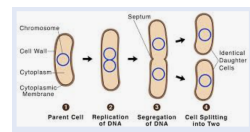
Adenine, Thymine Cytosine, Guanine, a *phosphate group* and a *sugar molecule Deoxyribose*, 50-60 atoms in a nucleotide.

The structural framework of DNA is defined by the sugar-phosphate backbone, which is *circular* for prokaryotic cells, and which is composed of alternating sugar and phosphate groups.

DNA is the queen in of the cell. All other molecules in the cell are present in multiple copies like chords of worker and soldier ants who serve the queen in an ant colony.

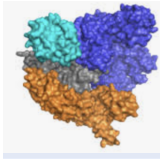
The information content of the genome of E-coli bacterium, $\sim 5 \cdot 10^6$ base pairs, amounts to thousand pages of English text.

These encode the *inheritable*, called *genetic*, information, which contains basic instructions/programs the cell needs to fulfil it's functions, where the essential functions of a prokaryotic cell are reproduction and metabolism and where the latter sustains the reproduction by *binary fission of cells*.



cell duplication

Proteins. A protein chain, also called called *polypeptide*, is a linear molecular concatenation of several hundred *amino acids* of 20 (sometimes 21 and rarely 22) types.



CAS9 protein, 0.01 mμ in diameter

(The information content in a protein is comparable to that of a few (3-6) short English sentences).

A protein is made of a chain, sometimes several (2-4) chains separately synthesised in a cell.

These *fold* into a specific closely packed three-dimensional shape – functional protein \mathcal{P} , which depends on particular sequences of the amino acids in the chains.

(This is remarkable! There is no a priori reason for a randomly looking linear concatenation of molecules to transform to a compact rigid structure.)

There are *few million* of protein molecules in a typical bacterial cell, where most \mathcal{P} are present in *a few thousand copies*.

RNA. These are molecular chains composed of four *ribonucleotides*, where the bases are similar to A,T,C,G. These chains, which serve several different functions in the cell, are assembled by templating segments of DNA.

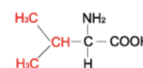
For instance, (*messenger*) mRNA molecules, on the average $\sim 1\ 000$ -long, serve as "scripts" for assembly of proteins.

The assembly is a performed by by *ribosomes* (which cooperates with other proteins and RNA) by a chemically elaborated process, called *translation*, which is, in truth, more than just a chemistry.

The information incoded the set of triples of letters in mRNA (of cardinality 64) is reduced in the course of translation by the *genetic code map* (with a loss of a bit of Schannon information, for $64 \sim 2 \cdot 20$) from the set $\{A, T, C, G\}^3$ to the set of twenty "amino-acid-letters" augmented by signs of "start" and "stop" instructions for translation.

Each amino acid according to this map is coded by several *codons* that are *triplets of the base-letters*. For instance the codons for *valine* are GTA, GTG, GCT.

Unlike how it is in the macroscopic world, the product protein \mathcal{P} , which is made according to the "script" written in the A,T,C,G-letters is ten times lighter than the weight of the letters in this "script", since nucleotides are about 3 times heavier than amino acids.



The number of *messenger* RNA in \mathcal{C} is a thousand time smaller than that of proteins and they also much shorter-lived.

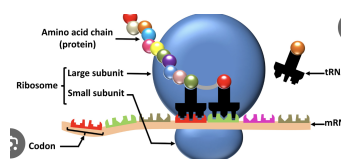
Valine
<0,001m μ

Universality of the Genetic Code. The genetic code, that is the *codone-to-amino acid* correspondence, is the same for all organisms except for some prokaryotes with small genomes and also *mitochondria* and *plastids* that are *organellas* (not classified as organisms), which reside inside eucaryotic cells and have independent DNA.

Ribosomes. Archaeal and bacterial ribosomes, $\sim 0,025\mu\text{m}$ in diameter, are composed of two asymmetric subunits.

The small subunit contains a 1500 nucleotides RNA, while the large one contains two RNA chains of 1500 and 3000 nucleotides. Besides there more than 50 proteins in there.¹⁴

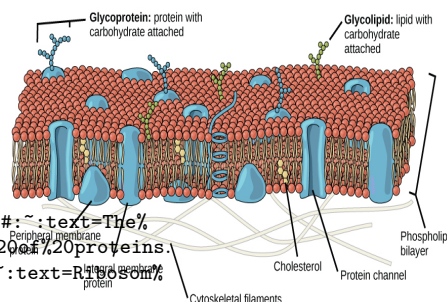
"The prokaryotic ribosomes contain 3 RNA strands and 52 protein subunits which can be divided into 1 RNA and 21 proteins in the small ribosomal subunit (aka the 30S subunit) and 2 RNA and 31 proteins in the large ribosomal subunit (50S subunit).Jan 3, 2021"



Ribosomes are complex tightly knit molecular motor-machines, which, amazingly, are assembled in the bacterial cells along elaborate biogenesis pathways with many intermediates in a couple of minutes¹⁵, where, the number of ribosomes in a fast dividing E-coli bacterium reaches 70 000.

Cell Membranes. The outer membrane sustains the *metabolic autonomy* (and individuality) of the cell by limiting diffusion of certain molecules in and out of the cell.

For instance H₂O, O₂, CO₂ and small *hydrophobic* molecules easily diffuse across phospholipid bilayer but the transport of ions and large molecules, such as sugars and amino acids relies on specific proteins embedded in the membrane.¹⁶



¹⁴<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3653068/#:~:text=The%20bacterial%20ribosome%20is%20a,amino%20acid%20sequence%20of%20proteins.>

¹⁵<https://www.nature.com/articles/s41467-023-40859-w#:~:text=Ribosome%20biogenesis%20is%20a%20complex,%2C4%2C5%2C6.>

¹⁶[https://www.ncbi.nlm.nih.gov/books/NBK9847/.](https://www.ncbi.nlm.nih.gov/books/NBK9847/)

Figure 2: *plasma membrane*

1.2 Sub-cellular Replicators: Viroids, Virusoids, Obelisks, Viruses, Satellites, Plasmids, Organellas, transposons, Prions and their Informational individuality

virus	size (nm)	genome size (base pairs)	genome type, capsid structure
porcine circovirus (PCV)	17	1,760	circular ssDNA, icosahedral
cowpea mosaic virus (CPMV)	28	9,400	2 ssRNA molecules, icosahedral
cowpea chlorotic mottle virus (CCMV)	28	7,900	3 ssRNA molecules, icosahedral
φX174 (<i>E. coli</i> bacteriophage)	32	5,400	ssDNA, icosahedral
tobacco mosaic virus (TMV)	40×300	6,400	ssRNA, rod shaped
polio virus	30	7,500	ssRNA, icosahedral
φ29 (<i>Bacillus</i> phage)	45x54	19,000	dsDNA, icosahedral (T3)
lambda phage	58	49,000	dsDNA, icosahedral (with tail)
T7 bacteriophage	58	40,000	dsDNA, 55 genes, icosahedral (T7)
adenovirus (linear DNA)	88-110	36,000	dsDNA, icosahedral
influenza A	80-120	14,000	ssRNA, roughly spherical
HIV-1	120-150	9,700	ssRNA, roughly spherical
herpes simplex virus 1	125	153,000	dsDNA, icosahedral
Epstein-Barr virus (EBV)	140	170,000	dsDNA, icosahedral
mimivirus	500	1,200,000	dsDNA, icosahedral
pandora virus	500x1000	2,800,000	dsDNA, icosahedral

Pospiviroidae and Avsunviroidae,
 cell organisms viruses proteins macromolecules Retro-
 transposons Edge of Life: Viruses, Obelisks(?), Prions
 Independent unitary Carriers of information that
 can influence Life processes
 Viruses and viroids are the simplest *self-replicating machines*.

1.3 Structural Genetic Individuality of Units of Life and Metabolic Autonomy of Cell, Organisms and Colonies of Organisms

The main *autonomous active players* in the game of Life are called (cellular) *organisms*; these, such as *bacteria, fungi, animals*, are composed of *cells*.

Organisms are *metabolically connected* and spatially/temporally *bounded*. (Well..., a network of 2000 years old Honey fungus in Oregon covers about 10 km²).

(A geometer may defines "organism" as a *maximal spatially connected* live entity where "maximal" means *not a part of another entity in this class*.)

Besides, there are certain *biologically active non-cellular* creatures, which replicate in host cells, and can be called *info-organisms*, such as *viruses, viroids, plasmids*.

The cell is an ensemble of (small) molecules, macro molecules (notably DNA, RNA, Proteins) and dynamic *macromolecular* structures enclosed in the *lipid bilayer (plasma) membrane*, which is about 5nm - 10nm thick. ($1nm = 10^{-9}m = 10^{-3}m\mu$.)

1.3.1 *Organisms, Life, Death.*

The basic *units of Life* are *individual organisms*; their lives are processes: an organism may be either *actively alive, dormant* or *dead*.

Besides there are seeds, spores, embryos and dried-out and/or frozen organisms, e.g dry Baker's yeast.

Life is accompanied by death, but bacteria and certain line cells, e.g. immortalised tumor cells, are virtually immortal.

1.3.2 Aging and programmed death

Death was invented by evolution viruses naked ras etc

1.4 Unit's Multiplicity Similarity, Universality, the Sources and the Roles of these

1.4.1 *Macromolecularity, Universality, Multiplicity.*

The structures and functions of all cells, which depend on carbon-based *macromolecules*, are closely *similar* on the basic level (in agreement with the idea *common ancestry of Life*).

There are five classes of these molecules: nucleic acids (DNA, RNA, $10^4 - 10^{10}$ Dalton), proteins $10^4 - 10^6$ Da, lipids (< 800 Da), ¹⁷ and carbohydrates. ¹⁸

All macromolecules, except for *chromosomal DNA*, are present in *many identical copies* in the cell.

left/right carbon bio-molecules (Pasteur)

(It may be significant that the number of atoms in a bacterium, about 10^{12} , is notably smaller than the number of bacteria in a cosy pond of water, say 10^{15} , because this makes an accidental extinction of a population of *exponentially growing* bacterial population in the "ideal world" is significantly smaller than an improbable creation of a viable bacterium.

If no *replicatively stable* organism could be made with $< 10^{18}$ atoms, Life couldn't have emerged as fast as it had happened on Earth.)

Potentially Exponential. multiplicity by doubling and

and linear multiplicity by Assembly Lines

Templating + Translation of 1-Dimensionally Stored Digital Information such as Replication of DNA followed by Cell Division, Templating of DNA and RNA, Assembly Line Production of Proteins Protein assisted assemblies, production of quasi similar organisms by Embryogenesis

explain!

Question. Does the complexity and/or multiplicity of *identical randomly looking* complex, static as well as dynamic, structures, such as programmed bio-heteropolymers. and/or the genetic code, and *stability* of a priori unstable structures in Brownian molecular environment is characteristic of Life?

For instance ...

the number of bacteria on Earth is estimated at 10^{30} and where majority appear in more than 10^{20} copies.

The high multiplicity in the Brownian environment is necessary for Life. to guaranty stability but probably(?) there. are non-live structures with such property

more delicate distributions of molecule numbers over cells and cells copy numbers

Examples: of simple generation of high multiplicity

Multiple granularity is A characteristic feature of Life on Earth (and of any conceivable chemically based life in the known physical Universe) is

¹⁷<https://www.ncbi.nlm.nih.gov/books/NBK26871#:~:text=The%20membrane%20lipid%20molecules%20are,phospholipids%2C%20cholesterol%2C%20and%20glycolipids.>

¹⁸<https://en.wikipedia.org/wiki/Polysaccharide#:~:text=Polysaccharides%20are%20major%20classes%20of,component%20of%20a%20plant%20cell.>

1.4.2 Multiplicity of Molecules in chemistry and Coherent Granularity in Biology and of Programmed Macromolecular Structures, Cells, Organisms, Colonies

universality of subcellular structures

Include 100 prisoners problem and how it is solved by multicellular organisms (cancer?)

Is high multiplicity of complicated structures possible outside life?

is life possible without multiplicity?

what is complexity?

snowflakes. turbulent patterns in the water flows

1.4.3 Conservation of Information and Reproduction of Structures

Organisms propagate by making (approximate) *copies* of themselves, which involves *molecular template replication* (of DNA), *binary fission* of cells and, for multicellular organisms, *embryogenesis*.

active entities viruses plasmids etc

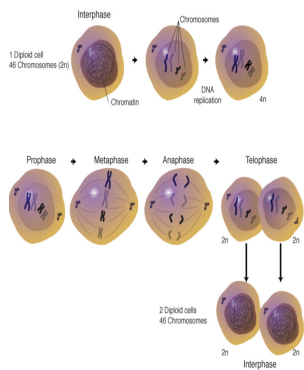


Figure 3: mitosis

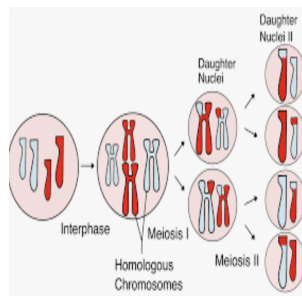


Figure 4: mitosis

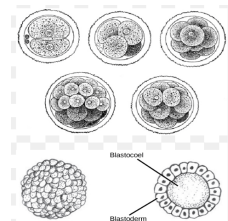


Figure 5: embryonic development

Eucariotic cells division proceeds in several steps (the *cell cycle*) in two different ways: *mitosis*, e.g. in you *somatic cells* when new tissues grow, and *meiosis* in the *gametes* (sex cells).

The typical time of of mitosis is between 2h and 24 h, but it may be 30 min in an early embryo where cells divide into smaller cells, which don't grow.

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¹⁹<https://www.ncbi.nlm.nih.gov/books/NBK9928/>

1.5 Controlled Metabolic Meta-Stability and Reactivity to (Symbolic)Signals

number of controle proteins in eukaryots, number signalling proteins

1.5.1 *Information and Signals*

. Besides flows of energies in cells there is a ceaseless exchange of signals between different parts of a cell, between different cells in an organisms and between different organisms.

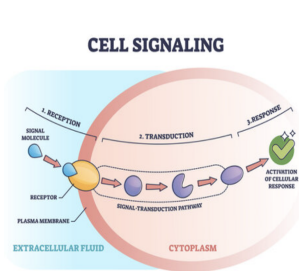


Figure 6: signal transduction



Figure 7: conversing in pheromones



Figure 8: memes transmission

1.6 Energy and Entropy Barriers, Catalysis, Enzymes, Multi-enzymatic Metabolism

Emergence of Life as a passage through an entropy barrier/labirint

1.6.1 *Enzymatic Homeostatic Metabolism.*

²⁰ Metabolism, an exchange of energy and matter with an *environment*, is accomplished by a network of (hundreds of different kinds of fast *enzymatic*) chemical reactions²¹ in an organism, which converse the *free energy* and matter from environment to the energy for running cellular processes and for the *synthesis* (and *degradation*) of molecules needed for continuous (re)building itself; "homeostatic" means *self-regulatory*.

Most of these reactions runs in *multiply repeated cycles*.

1.6.2 Ensembles of Enzymes

different speeds of enzyme lead to different end results of reactions

without enzymes even slow life would be impossible in a way these speeds encode the result

1.6.3 Programmed matter/energy flows and Physical/Chemical realisation of Information flows

Spontaneous versus programmed (multi-enzymatic) Chemistry and self assembly

sequential molecular assembly

Degradation and digestion

Specificity and universality of enzymatic Chemistry

1.7 Brownian Spontaneous and Programmed Self-Assembly Including Protein Folding

spontaneous assisted

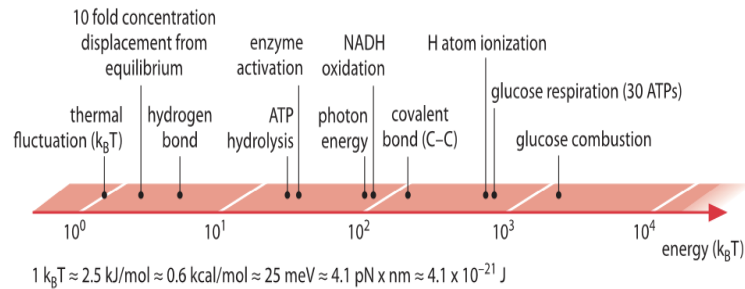
and programmed self-assembly; CRISTALLS MEMBRAINS, PROTEIN STRUCTURES RNA SECONDARY COMPLENETARYTY-CRICK WATSON. DNA helix. amiloids, prions

²⁰<https://www.oatext.com/the-basis-of-metabolic-homeostasis-demand-regulated-energy-metabolism.php#:~:text=Metabolic%20homeostasis%20is%20determined%20by,by%20glycolysis%20and%20oxidative%20phosphorylation.>

²¹<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6943030/>

1.7.1 Energy, Symmetry, Crystals and Crystallization

1.7.2 Micelles, Membranes and Protein Assisted Self-assembly



1.7.3 Micelles, Membranes and Protein Assisted Self-assembly

Cells: membranes, For instance H_2O , O_2 , CO_2 and small *hydrophobic* molecules easily diffuse across phospholipid bilayer but the transport of ions and large molecules, such as sugars and amino acids relies on specific proteins embedded in the membrane.²²

1.7.4 From Polypeptides to Proteins

Logic of protein folding: map from sets of energy parameters to coordinates of the minima points

1.7.5 Nuclear Acids Self-Assembly by Crick Watson Complementarity and DNA Computing

1.8 Combinatorics of Time/Space/Function Interaction/Relation Networks between Units of Life

hierarchy of similarities (network)

different levels of connectivity in biology (including: mecano/chemical, functional ... ancestor, by sex repr in progeny, horizontal gene transfer)

²²<https://www.ncbi.nlm.nih.gov/books/NBK9847/>.

1.8.1 Plant communication

(co)functional network
causation network
energy/food network
planets are separated by empty space but cells are
by the walls

1.8.2 Many Forms of Connectivity in Networks of Life

predation and parasitism
mechanical/chemical, functional ... ancestor-tr, by
sex repr in progeny, horizontal gene transfer

1.8.3 Ecological Networks

.
Most organisms on Earth (estimated 10^{30} are *unicellular*: bacteria, yeast, amoeba..., amount to $\approx 10\%$ of the carbon (estimated 10^{10} ton) biomass on Earth.

(The bulk of bio-carbon, estimated $45 \cdot 10^{10}$ ton, is accumulated in plants, 2% in fungi and less than 0.5% in animals.²³)

The *number of cells* in an animal varies from only a few in some *myxozoan parasites*²⁴ to about 10^{17} in a blue whale.

<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC9986037/>

<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC9986037/>

In humans, cell volume varies across more than five orders of magnitudes, ranging from sperm cells of about $30 \mu\text{m}^3$ (BioNumber database (Milo et al. 2009), BNID 109891, 109892) and red blood cells of about $100 \mu\text{m}^3$ (BNID 107600), to fat cells of $600,000 \mu\text{m}^3$ (BNID 107668) and egg cells of $4,000,000 \mu\text{m}^3$ (BNID 101664).

10^{30} living bacteria, 30,000 formally named species and 10^{31} phages

about 10^{12} atoms in bacteria and about 10^{12} bacteria of 1000 species in human guts

plasmids and in bacteria. mitochondria in eucariotic cells

. Today our planet is populated by about a trillion species of microorganisms [3]. They include bacteria - the vast majority - but also viruses, fungi, and unicellular microorganisms. At any one time there are

²³[https://en.wikipedia.org/wiki/Biomass_\(ecology\)](https://en.wikipedia.org/wiki/Biomass_(ecology)).

²⁴<https://earthlife.net/myxozoa-tiny-cnidarian-parasites/>.

roughly five million trillion trillion (10³⁰) living bacteria and 10³¹ phages (viruses that infect bacteria) that attack them, killing 40

There are multiple material and signalling interactions between living entities on Earth, for instance:

rabbits eat grass and foxes eat rabbits,

bumblebees pollinate plants,

most plants obtain 80% of phosphorus from *mycorrhizal fungi* (which increase the absorption by the roots) and deliver about 20% of their *photosynthesized carbon* to the fungi.²⁵

The main structural entities of *Life on Earth*, (the only Life we know of) with different kind of connections between their constituents are

macromolecules → *cells* → *organisms* → *colonies&communities* → *populations* → *ecologies*

with a variety of connections between their constituents are where organisms are composed of cells and cells

as follows

programmed heteropolymers → *macromolecular complexes* → *cells* → *organisms*

→ *colonies* → *communities* → *populations* → *ecologies*

and the main "flows" are

1.9 *Evolution, Selection Speciation.*

Evidence of bacterial life on (4.5 billion years old) Earth goes back to 3-4 billion years ago. <https://www.ncbi.nlm.nih.gov/books/NBK9841/#:~:text=The%20eukaryotes>

Eukarya. (<https://www.ncbi.nlm.nih.gov/books/NBK9841/#:~:text=The%20eukaryotes>) Following about a billion years prokaryotic evolution, a merger of an archaean and an aerobic bacterium created the highly *compartmentalized* eukaryotic cell with the membrane-bound nucleus.

Multicellular organisms (today, these are plants, fungi and animals) came to the scene about 1.5 billion years later and the major radiation of animal phyla (Cambrian explosion) was recorded in fossil dated 530-540 million years ago.

²⁵<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC10369077/>

The driving force behind evolutionary diversification is the *exponential growth* of uncontrolled reproduction (think of Fibonacci rabbits²⁶), where "*exponential*" is cut-off to "*linear*" by mortality of progeny in a realistically limited environment.

Then successive quasi-exponential *amplification of rare events* – fixation of lucky (possibly not only advantages) mutations in populations, makes possible *evolution by the natural selection*.

Since this was identified in the Darwin and Wallace 1858 paper, the evolution is described in terms of *hierarchical* "selection gradient" (fitness) that is the (highest semi-locally time averaged) reproduction rate of species *X* under constrain imposed by (random and variable) *environment*, where this "environment" includes the presence of other species (e.g. parasites) as well as the population density of *X* itself.

(Thus you see why a tree needs to grow tall and build the otherwise useless trunk composed of dead tissue.

But *sexual selection*, for instance, doesn't so nicely fit into this picture.²⁷

inventions evolution

1. Multicellularity
2. The eye and ear muscles and nervous system

1.9.1 Reception and Production of Information by Nervous System

All animals have a true nervous system except sea sponges. Cnidarians, such as jellyfish, lack a true brain but have a system of separate but connected neurons called a nerve net. Echinoderms, such as sea stars, have neurons that are bundled into nerves.

https://en.wikipedia.org/wiki/List_of_animals_by_number_of_neurons

immune system

3. The brain
4. Language
5. Photosynthesis
6. Sex
7. Death
8. Parasitism
9. Superorganism
10. Symbiosis

²⁶ *Liber Abaci*, 1202 https://en.wikipedia.org/wiki/Liber_Abaci

²⁷ https://en.wikipedia.org/wiki/Sexual_selection,
<https://www.pnas.org/doi/full/10.1073/pnas.0901129106?doi=10.1073%2Fpnas.0901129106>,
<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2176171/#:~:text=The%20lek%20paradox>.

What are the major evolutionary innovations? Evolutionary innovations—the bony skeleton of vertebrates, avian flight, or the insect pollination system of angiosperms, for example—have in recent years become the focus of much fertile new research in evolutionary biology.

muscles and the nervous system

Evolutionary Innovations - The University of Chicago Press
The University of Chicago Press

PMC 4547294. PMID 25838283. Koonin, Eugene V. (19 August 2016). "Viruses and mobile elements as drivers of evolutionary transitions". *Philosophical Transactions of the Royal Society B: Biological Sciences*. 371 (1701): 20150442.

<https://www.newscientist.com/article/dn9951-top-10-lifes-greatest-inventions/>
[https://www.sciencedirect.com/science/article/abs/pii/S016895251100093X?](https://www.sciencedirect.com/science/article/abs/pii/S016895251100093X?via%3Dihub)
via%3Dihub

<https://www.liebertpub.com/doi/full/10.1089/ast.2021.0119>

Oxigene breathing. photosiyt ammonia

1.9.2 units of Evolutions and Initial stability of (sequences of) successive generations

(of populations) under slow variation of the environment and variability on the (evolutionary) long time scale

Lamarckian interpretation of Luria Delbruck- experiment

1.10 Cloudy objects of evolution, kind of Quasi[species of Manford, Evolutionary Designed Passive Stability, Learning and Active Adaptability

Organisms are *stable* under small (and not so small) variation of the *environment*.

Passive Examples.

The soft body of an octopus can fit into all kinds of nooks and crannies.

Stability of bipedal locomotion is facilitated by elasticity of the human vertebral column.

Active Examples.

Growth of calluses protects the skin on your hands and on the soles of your feet.

Increase of the blood flow to the gastrointestinal *system facilitates digestion*.

Learning by the nervous system enables *behavioral*

plasticity of animals, from worm *C. elegans*²⁸ to humans.

1.11 What is Life Answers by Astrobiology, Physics, Chemistry and Computer Science

II. *Astrobiological Definition* (NASA following Carl Sagan). Life is a self-sustained chemical system capable of undergoing Darwinian evolution.²⁹

(Sounds funny: does scrutinizing signs of Life on a remote planet needs a D-detector for guarding against outcomes of non-Darwinian evolutions?)

³⁰

1.11.1 Entropy and Free Energy Redistribution

1.11.2 Triggering by Signalling as instability

1.11.3 Positionality Chemistry opposite to Physical/mathematical quantification of Lavoisier

1.11.4 Catalysis, Autocatalytic Networks and Replication

1.11.5 Automata, Von Neumann Universal Constructors, Artificial Life

What pays the role of matter/energy flows in computer modes of life (low levels of information structures)

I. SCHRÖDINGER 1944.³¹

...[an organism] *can only keep ... alive, by continually drawing from its environment negative entropy— which is something very positive... .*

the most essential part of a living cell - the chromosome fibre may suitably be called an aperiodic crystal.

*An organism is a thermodynamic system with an organized molecular structure that can reproduce itself and evolve as survival dictates*³²

²⁸<https://pubmed.ncbi.nlm.nih.gov/15296777/#:~:text=The%20nematode%20worm%20Caenorhabditis%20elegans,and%20its%20own%20physiological%20state..>

²⁹[https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3937540/.](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3937540/)

³⁰Fermi paradox: maybe something else?

³¹WHAT IS LIFE?What Is Life? The Physical Aspect of the Living Cell.

<https://www.arvindguptatoys.com/arvindgupta/whatislife-schrodinger.pdf>

https://en.wikipedia.org/wiki/What_Is_Life%3F

³²<https://en.wikipedia.org/wiki/Life>.

"Thermodynamic system" is *a body of matter and/or radiation separate from its surroundings that can be studied using the laws of thermodynamics*.³³

These laws, however, don't tell much about life and an elegant mathematical axiomatization of these laws by Elliott H. Lieb and Jakob Yngvason³⁴ doesn't to understand Life help either.

In fact, physics (thermodynamics especially) is all about *generic-by-symmetric* objects and phenomena, while Life is something physically *exceptional* one. (We'll explain this later on.)

An honest physicist's definition reads:

Life is a physical world phenomenon, which is indescribable entirely in the language of physical laws.

Imagine, for instance. you find a functional Watt-style engine on the icy surface of Enceladus – a small moon of Saturn. It will take no long to understand how it works, but there will be no explanation of what brought it there in terms of the thermodynamics laws.

Fermi paradox: maybe Life is something else?

<https://www.mdpi.com/2075-1729/11/4/308>

<https://www.nature.com/articles/s41557-023-01276-0>

III.CHEMISTS: ... *Life is a selfsustaining kinetically stable dynamic reaction network derived from the replication reaction.*

*Biology is just an elaborate extension of replicative chemistry*³⁵ that is an *autocatalytic* (replication) network, where a set of molecules catalyzes an identical second set.³⁶

In the early 1970s, this idea was articulated in several ways.

Eigen: dynamics of *Hypercycles*.³⁷

Gánti: construction of *chemotons*.³⁸

³³<https://en.wikipedia.org/wiki/>

³⁴<https://www.sciencedirect.com/science/article/pii/S0370157398000829>.

³⁵<https://hal.science/hal-01003174/document>.

³⁶CATALYSIS: THE SECRET TO LIFE?<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6315399/>.

³⁷M. Eigen, P.Schuster, The Hypercyde, A Principle of Natural Self-Organization https://metabolic-economics.de/pages/seminar_theoretische_biologie_2007/literatur/schaber/Eigen1977Naturwissenschaften64.pdf.

<https://www.sciencedirect.com/topics/mathematics/hypercycles>.

³⁸The Principles of Life reviewed by Gert Korthof,<https://wasdarwinwrong.com/korthof66.htm>

Maturana&Varela: philosophy of *autopoietic* (self-repairing) systems.³⁹

Rosen's algebra of *(M,R) systems*⁴⁰

Kauffmann: dynamics of *collectively autocatalytic* sets.⁴¹

For instance, according to Tibor Gánti,⁴² the essentials of Life – *metabolism, self-replication, and a bilipid membranecan* be chemically implemented as follows.

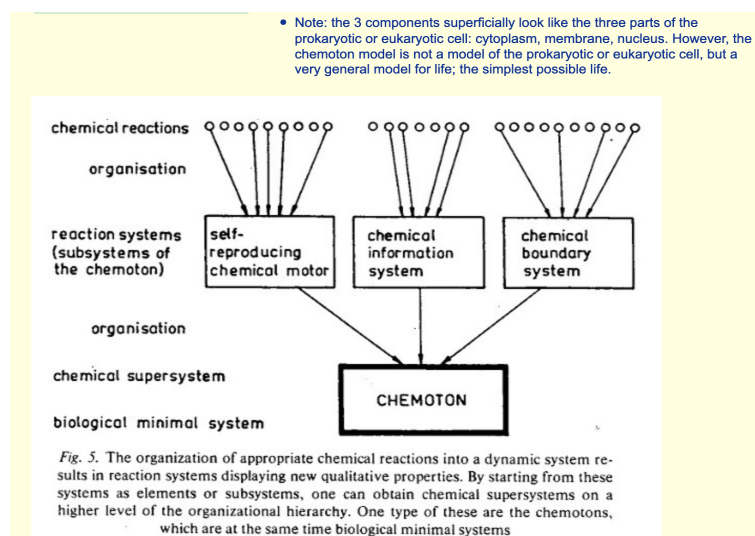


Figure 9: Gánti's chematon

PROBLEMS WITH CHEMISTRY. It is an overstatement saying that DNA replication is a kind of "autocatalysis".

Not even transcription (and even less so translation) is just "a chemical reaction catalyzed by RNA-polymerase" (ribosome for translation); by saying so, one disregards the iterative and the mechanical aspects of the process.

Neither *programmed self-assembly*, e.g. protein folding can be described in purely chemical terms.

Life is derived from chemistry but, it is not "just chemistry", not even on the sub-cellular level.

Just saying that DNA-replication is "a chemical

³⁹Autopoiesis and Cognition(Book) https://monoskop.org/images/3/35/Maturana_Humberto_Varela_Francisco_Autopoiesis_and_Cognition_The_Realization_of_the_Living.pdf

⁴⁰<https://yannickprie.net/archives/ENACTION-SCHOOLS/docs/documents2008/Autop.Rosen.pdf>.

⁴¹<https://royalsocietypublishing.org/doi/10.1098/rsif.2018.0808>.

⁴²The Principles of Life reviewed by Gert Korthof,<https://wasdarwinwrong.com/korthof66.htm>.

reaction catalyzed by DNA-polymerase" ignores reveal the mechanical aspect of this process.

Non-existence of a (unique comprehensive) mathematical definitions for "universums" : *Algrithm, Life, natural language, universal self reprodoction real universe* Neither there is a precise universally applicable concept of equivalence between partial definitions, e.g. between Turing machine and recursive functions.

IV. LIFE AS A COMPUTATION

Von Neumann universal replicator stable replication in Brownian enviroment

Relational biology Rosen (M,R) systems, modularity⁴³

Vallient PAC algorithms⁴⁴

The algorithmic origins of life Sara Imari Walke and Paul C. W. Davies⁴⁵

Algorithms in nature: the convergence of systems biology and computational thinking Saket Navlakha and Ziv Bar-Josepha,⁴⁶.

A New Replicator: A theoretical framework for analysing replication Zachar&Szathmàry ⁴⁷

=====
<https://plato.stanford.edu/entries/teleology-biology/>

Non-existence of a (unique comprehensive) mathematical definitions for "uni- versums" : *Algrithm, Life, natural language, universal self reproduction real universe* Neither there is a precise universally applicable concept of equivalence between partial definitions, e.g. between Turing machine and recursive functions.

But *we must know, we will know.*⁴⁸

NO: we can't realistically (in the span of hundred years) decide which two pages programs on your laptop computer will terminate within 10⁹ years and which will run for 10¹⁰⁰ years, nor can we, in general, recover such a program from the outcome of its performance.

⁴³<https://bmcsystbiol.biomedcentral.com/articles/10.1186/1752-0509-7-128>.

⁴⁴<https://people.seas.harvard.edu/~valiant/evolvability-2008.pdf>.

⁴⁵<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3565706/>.

⁴⁶<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3261700/>

⁴⁷<https://bmcbiol.biomedcentral.com/articles/10.1186/1741-7007-8-21>.

⁴⁸David Hilbert.

1.12 Mathematician's Perspective on Life

1.12.1 Biological and non-Biological Mathematical Patterns of Life

May

Haldan etc

Turing equation

Non-existence of a (unique comprehensive) mathematical definitions for "universums" : *Algorithm, Life, natural language, universal self reprodoction real universe* Neither there is a precise universally applicable concept of equivalence between partial definitions, e.g. between Turing machine and recursive functions.

MATHEMATICIAN: Search for *mathematically structured patterns* in Life and for *mathematical interpretations of basic biological concepts and phenomena*, e.g. expressed by the italicized words in the above **1-12**.

What is percentige of conceivable bacterial genome sequences $c 4^{1000000}$ = (properly understood genomes are below 10^9)

This may or may not be applied to Life on Earth that is an outcome of such a program, call it Life-0, which has been running for almost four billion years.

The accumulated knowledge of the performance of Life-0 by biologists, moist likely, uniquely define Life-0 and this program must be short and simple.

And although a full determination of Life-0 is hardly possible, fragments of this program can be seen trough the keyhole of mathematics, If we switch the focus from this keyhole next to our eye to **Life Phenomena**, which are far from a mathematician's mind eye.

⁴⁹

Math for its own beauty inspired by by "Life" technically useful math for biologists as *judged by biologists*

patterns of Life for their hidden math beuty

patterns of Life just for their own sake

aspects of Life essential for our surviaal

The clubs we attend and the ones we don't

CONDORCET, MALTUS, DARWIN, TREE OF LIFE AND LIMBIC LANGUAGE

[https://www.thelancet.com/journals/lancet/article/PIIS0140-6736\(05\)](https://www.thelancet.com/journals/lancet/article/PIIS0140-6736(05)77853-3/fulltext)

77853-3/fulltext

Scietific mathematics (Condorcet, Malthus, Darwin, Poincare, Turing, Thom. Vision of the "real world as described by sciences Physics, Chemistry Biology Linguistic +? in the light of Math)

⁴⁹You may not believe it, but not all non-mathematics is nonsense. Casually saying "... a protein composed of a few thousand amino acids..." or "... the Brownian motion discovered by Brown...", is as silly as a non-mathematicia

Malthusian e^z via the universal covering of $S^1 \subset \mathbb{C}^\times$

A mathematician reads the newspaper : Paulos,
John Allen
modularity granualirity
different levels of connectivity in biology (including. mecanico/chemical, functionsl ... ancestortr, by sex repr in projeny, horizontl gene transfere

UNNATURAL COMBINATION OF RANDOM AND NON-RANDOM

("Reduction of Entropy")

Basic example:

Repetition of random patterns.

*The striking regularity with which the same hybrid forms
always reappeared whenever fertilisation took place
between the same species...*

("Mendel Law " 200 000, "Hardy Weinberg principle" 100 000, "Hardy theorem". 10 000.)

In pysical. systems/models "random" perturbs "non-random".

By repeating simple actions Life built non-random systems out of random material.

Hundred prisoners problem.

"Two balls making π " problem

15:16 Why do colliding blocks compute pi? because the mass-weighted sum of squares doesn't change

Von Neumann universal constructor.

Why there is no mathematical definition of "life" of "universal constructor" or of an "algorithm"

What are algorithms implemented and implementable by Life systems including our mathematical brains

1.12.2 Information in Life

1.12.3 Probability /Life

1.12.4 Future Mathematics: Biochemistry but not yet Biology

trees of cosmic rays

1.13 Summary of words and ...

biologist's words+ regularity, multiplicity, improbability, stochastic symmetry breaking, stochastic stability, selective instability, key-lock complementarity, self-assembly, improbability, free energy, applicability range, reappearance, numbers, networks, coded signals, language, metaphors, grammar, symbolic, arbitrary, algorithms, category theory, products spaces, quotient, subquotient signal/information exchange history dependent frozen accident teleology entropic tunnels and labirinth desined iterative instability .

FOURTEEN (SEMI-FORMALLY FORMULATED) APPARENT (MUTUALY INTER-DEPENDENT) FEATURES OF THE "FLOWS OF LIFE ON EARTHS", WHICH ARED NOT PRESENT IN PHYSICAL AND CHEMICAL SYSTEMS.

I. *Improbably high multiplicity* of a presence of *nearly identical logically complicated* structural/functional entities.

II. *Metabolic stability* (homeostasis and adaptation) under *generic* (stochastic environmental) perturbations with an *abrupt* breakdown (transition to death) beyond certain threshold.

III. High sensitivity to *specific energetically insignificant* informative *signals*.

IV. Metabolic *autonomy* and *info-individuality* of cells.

V. *Enzymes dependent* quasi-cyclic cellular metabolism.

VI. *Static one dimensional digital* storage of the (basic) inheritable information in the cells.

VII. *Low error rate template* molecular replication and *binary self-reproduction* of cells.

VIII. *Digitally programmed production line* molecular *assembly* and *repetative* (macromolecular) *granularity*.

IX. *Programmed molecular self-assembly*.

IX. Micro-scale structural and functional *universality*.

XI. Macro-scale vertical and horizontal (space/time) *connectivity*.

XII *Recursive multiple* self reproduction of organisms and "quasi-organisms" (e.g. viruses).

XIII *Initial stability* of (sequences of) successive generations (of populations) under slow variation of the environment and *variability* on the (evolutionary) long time scale.

<https://www.sciencedirect.com/science/article/abs/pii/S0959440X23000386>

Diverse mechanisms of polysaccharide biosynthesis, assembly and secretion across kingdoms

XIV. *Improbable structurality. and specificity* of individual Life related entities and phenomena rep-

resented in high dimensional. parameter spaces of physically possibilities.

1.14 Brain, Mind, Language and Ergo

2 Anthropocene

3 Molecular Machines and Bio-engineering

4 topics

////////////////////////////////////v////////////////////////////////////
ADD

resolvable specificity of trees versus universality of general networks

definition of sublevel trees

bacterial+viral systems

Non-existence of a (unique comprehensive) mathematical definitions for "universums" : *Algorithm, Life, natural language, universal self reproduction real universe* Neither there is a precise universally applicable concept of equivalence between partial definitions, e.g. between Turing machine and recursive functions.

Australian fungi in UV light

Impossibility of "rigorous" math outside common language

different reference examples(phenomenology) in different sciences math sci Physics: both strings and sand dust

energy of covalent bonds(to molecules ??) mathematics of catalyses

modularity, granularity

different levels of connectivity in biology (including. mecanico/chemical, functionsl ... ancestortr, by sex repr in projeny, horizontl gene transfere

4.1 *Terminology and Rudiments of Cellular and Molecular Biology*

4.1.1 Organisms and Cells

Eukarya includes:

single-celled "protista", e.g. *algae, amoebas, plasmodiums,*

fungi, e.g. molds, yeasts, mushrooms,

plants, e.g. mosses, ferns, conifers, flowering plants,

animals, e.g. corals, worms, insects, humans,

Eukaryotic cells greatly vary in size and *the number of cells* in an animal also vary: from only a few in *somemyxozoan parasites* to about 10^{17} in a *blue whale*.

Binary Fission Procariots and sometimes protists reproduce by binary fission

A bacterium in a nutrient medium may divide every 20min, thus making 10^{15} copies of itself in 24 hours, a "blob" 10cm in size and 10^{30} copies in 48 hours.

This is equal to an estimated number of bacteria on Earth, ⁵⁰ that could cover the whole surface of Earth by 20mm thick film.

Small hydrophobic molecules and gases like oxygen and carbon dioxide cross membranes rapidly.

which means that some molecules can diffuse across the lipid bilayer but others cannot. Small hydrophobic molecules and gases like oxygen and carbon dioxide cross membranes rapidly.

Cell are the menr units where the

that contains the fundamental molecules of life and of which all living things are composed

4.1.2 Photosynthesis and Cellular respiration

RuBisCO <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC7610757/#:~:text=Ribulose%2D1%2C5%2Dbisphosphate,in%20the%20global%20carbon%20cycle.> and

Chlorophyll P680 absorbs light energy and transfers resonance energy in the antenna complex, ending in the reaction center where specific chlorophyll P700 is located.

That plants receive some energy from light — in addition to air, soil, and water — was first discovered in 1779 by Jan Ingenhousz.

4.2 *Digital Storage of Information, Genes*

. Basic instructions a cell needs to sustain itself are "written" (in four small "molecular letters" A,T,C,G)

⁵⁰[https://www.ncbi.nlm.nih.gov/pmc/articles/PMC101423/.](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC101423/)

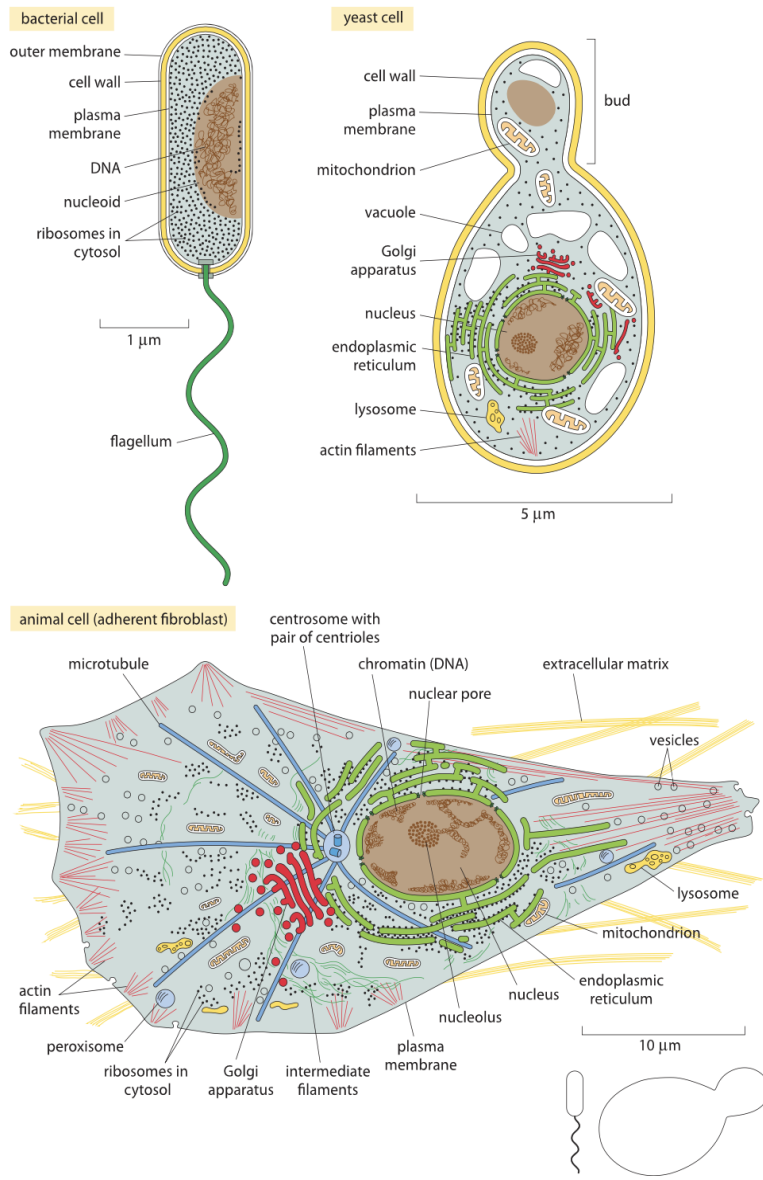


Figure 5: The Standard Cells. (A) A bacterium revealing its characteristic size and occupancy. (B) A yeast cell showing its characteristic size, its organelles and the number of various classes of molecules present within it. (C) an adherent human cell. We note that these are very simplified schematics so for example, only a small fraction of ribosomes are drawn etc. (Bacterium and animal cell adapted from B. Alberts et al., *Molecular Biology of the Cell*, 5th ed., New York, Garland Science, 2008)

Figure 10: Three types of cells

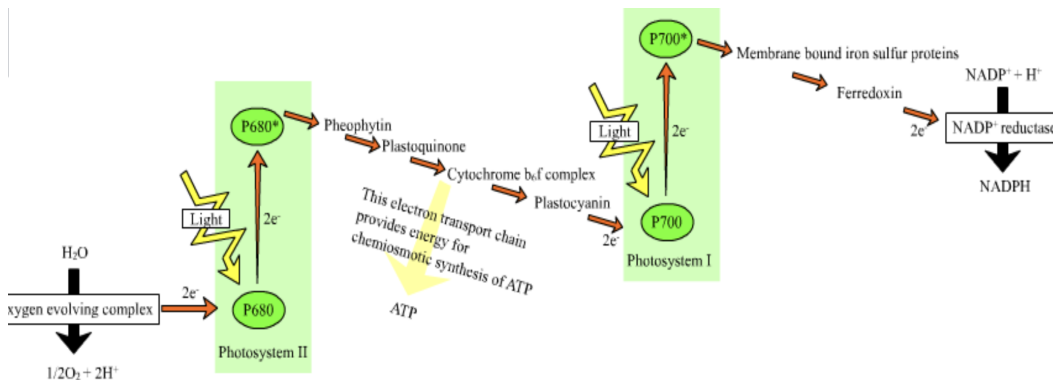


Figure 11: fragment of the metabolic network of a cell

on a (linear or circular) polymer molecule called DNA (or on several such molecules).
 The strings in these letters are, (sometimes only partially and sometimes ambiguously) *segmented*.

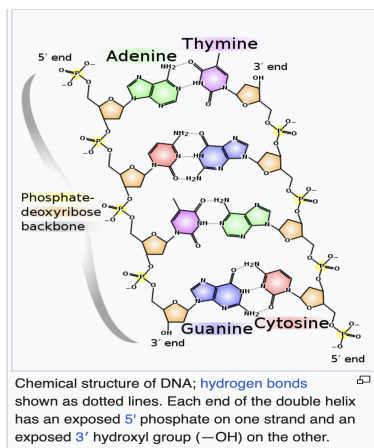


Figure 12: DNA: Chemistry

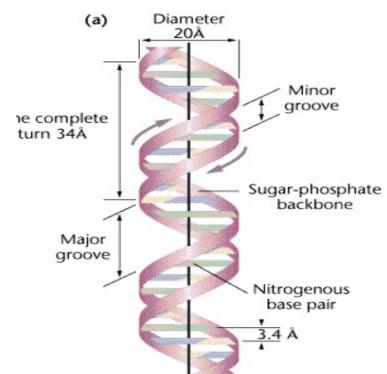


Figure 13: DNA: geometry

4.3 Information Decoding: Protein Self-Assembly by Folding

. DNA-sequences IN ATCG letters are "*translated*" (with an intermediate transcription to mRNA) to sequences in 20 (sometimes 21 and 22) "letters" represented by 20 (21,22) amino acid residues.

Then, guided by specific polarization of these residues and certain affinities between them, such a chains

folds (some don't) into a particular compact 3-dimensional protein body, needed for special functions of this in the cell.

In general, a protein may be composed of several chains and often several proteins self-assemble into *protein complexes*, e.g. protein shells of viruses.

4.4 *Taxonomy: Multiplicities of Similarities*

. Species of living organisms on Earth are represented, at least historically, by many very similar individuals. (It is *very* many for unicellular organisms, especially for bacteria, and also for some multicellular ones, e.g. for modern humans.)

Besides there is a (not always) unambiguous, *hierarchical clusterization* of organisms according to their morphological, genetic and ancestral similarities.

TRADITIONAL TAXONOMIC NOMENCLATURE.⁵¹ (This is different for prokaryotes – Bacteria&Archaea – and even more so for viruses)

$$\begin{aligned} & \textit{individuals} \in [\textit{species}] \in [\textit{genera}] \in [\textit{families}] \in [\textit{orders}] \in [\textit{classes}] \\ & \in [\textit{phyla}] \in [\textit{kingdoms}] \in [\textit{domains}] = \{ \textit{Bacteria}, \textit{Archaea}, \textit{Eukarya} \} \end{aligned}$$

Examples. (i) There are 36 (recognized) phyla in the animal kingdom, where the most common. are

- Annelida (worms, leeches)
- Arthropoda (insects, spiders, crustaceans)
- Chordata (mammals, fish, reptiles, birds)
- Cnidaria (jellyfish, anemone, corals)
- Mollusca (octopuses, squid, cuttlefish)
- Platyhelminthes (flatworms, tapeworms, flukes)
- Porifera (freshwater sponges, sea sponges).

(ii) There are over 600 primate species and subspecies recognized by IUCN)⁵² with over 99.99% of primate biomass accumulated in humans.

4.4.1 **Recursion, Trees, and Fractality in Life**

trees of cosmic rays

Why does then Nature enjoy expressing yourself in the images of trees, why does she do it so consistently, so much unproportionally frequently with respect to the modest share of the trees among general networks?

⁵¹<https://kids.britannica.com/students/article/biological-classification/611149>.

⁵²<https://en.wikipedia.org/wiki/Primate>

Phylogenetic Tree of Life

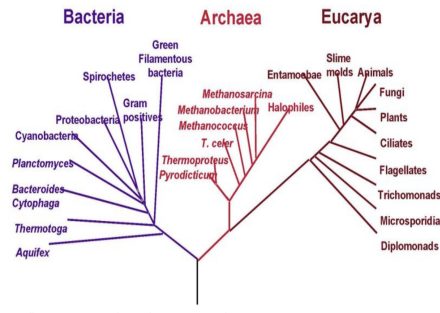


Figure 14: real tree.

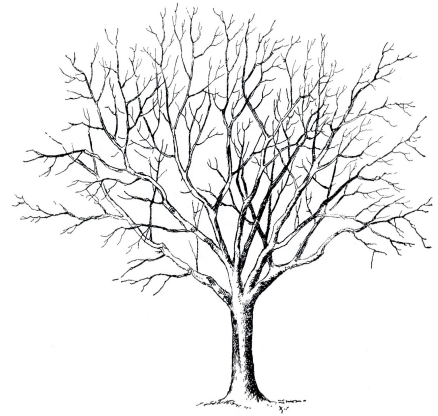


Figure 15: schematic tree.

The apparent reason for this is one-dimensionality of time versus three-dimensionality of space: the trees in the above pictures are still images of time dependent processes of spacial growth and development cast in unmovable space.

But more often than not, when the ancestral patterns are not physically preserved and only the final stage of a developmental process is recorded by Nature, the beauty of trees is still there, but now visible only with an imaginative mathematician's eye.

The most famous is the phylogenetic tree of Life worth seeing first on the grand scale

entropically it is easier to diverge than converge

Marvellous branching patterns of trees repeatedly come again and again everywhere in nature.

In arrangements of leaves and of veins in the leaves of trees.



Figure 16: fern.



Figure 17: veins in leaves.

In the neurones and in the lungs in your body.

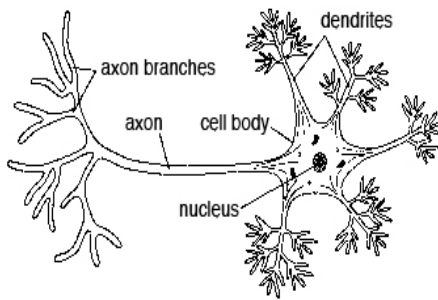


Figure 18: schematic neuron.

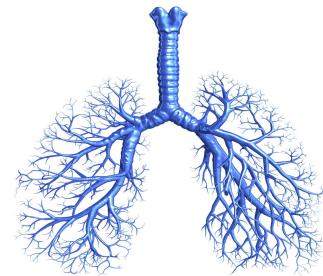


Figure 19: schematic lungs.

On many scales in your vascular, lymphatic and nervous systems.

Also in non-living things: in growing crystals, in

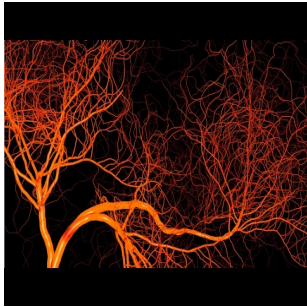


Figure 20: Blood Vessels

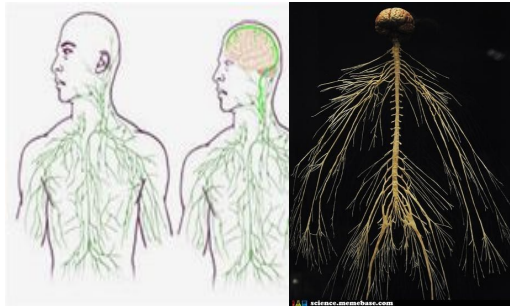


Figure 21: lymphatic system Figure 22: nervous system

branching of electrical discharges, in the drainage systems of rivers.



Figure 23: fractals in agate

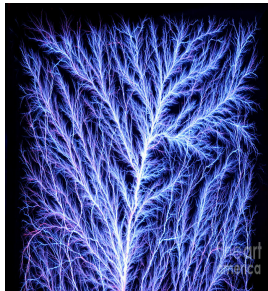


Figure 24: electrical discharge



Figure 25: drying Desert River

For a mathematician, trees are particular (connected) *networks* also called (connected) *graphs*, namely the ones *without cycles*.

It takes a long and involved discussion, which goes well beyond what mathematicians call *graph theory*, to fully and truly explain why "no cycles" makes trees so special.

And even if you have no idea why this is so, why trees are special, you can predict this by following an amazingly effective mathematician's motto:

If you don't understand – **COUNT!**

Indeed, simple – this is simple if you are a mathematician – counting shows that there are by far more networks in the world (of mathematics) than of trees

which are bare of cycles:

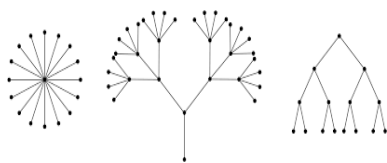


Figure 26: trees

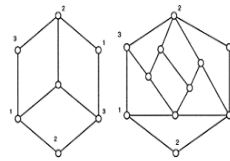


Figure 27: networks with cycles

the number of different graphic trees with N nodes is of the same order of magnitude as the number 2^N of binary sequences of length N , while the number of general graphs with N nodes is of order $2^{\frac{N^2}{2}}$ – this is incomparably greater than 2^N .

To appreciate the difference between these numbers think of something large, say of the synaptic memory system of you brain, or of the electronic memory of a powerful computer. However huge, they hardly contain more than $2^{50} \approx 10^{15}$ (million billions) units – each unit is capable to store a single bit of information. But this makes only a tiny fraction of the number $2^{25 \times 50} = 2^{1250}$.

(Even the number of atoms in the entire (observable) universe, something which is evaluated close to 2^{300} , is dwarfed by the enormity of this $2^{1250} \approx 10^{375}$.)

Why does then Nature enjoy expressing yourself in the images of trees, why does she do it so consistently, so much unproportionally frequently with respect to the modest share of the trees among general networks?

The apparent reason for this is one-dimensionality of time versus three-dimensionality of space: the trees in the above pictures are still images of time dependent processes of spacial growth and development cast in unmovable space.

But more often than not, when the ancestral patterns are not physically preserved and only the final stage of a developmental process is recorded by Nature, the beauty of trees is still there, but now visible only with an imaginative mathematician's eye.

The most famous is the phylogenetic tree of Life worth seeng first on the grand scale

and then looked at closer at the branches of our own "family trees".

And if, myopically, you focus on yourself, you'll see your personal family tree which looks, at least for a mathematician's eye, quite different: the directionality is reversed and the tree branches in both time

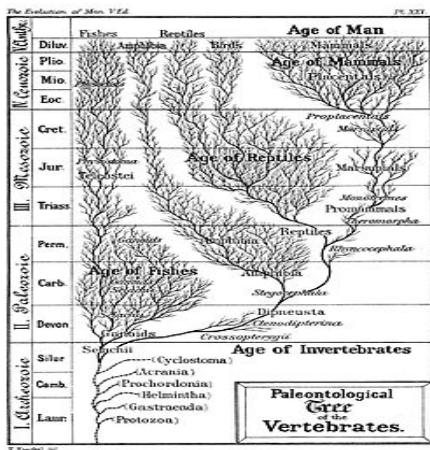
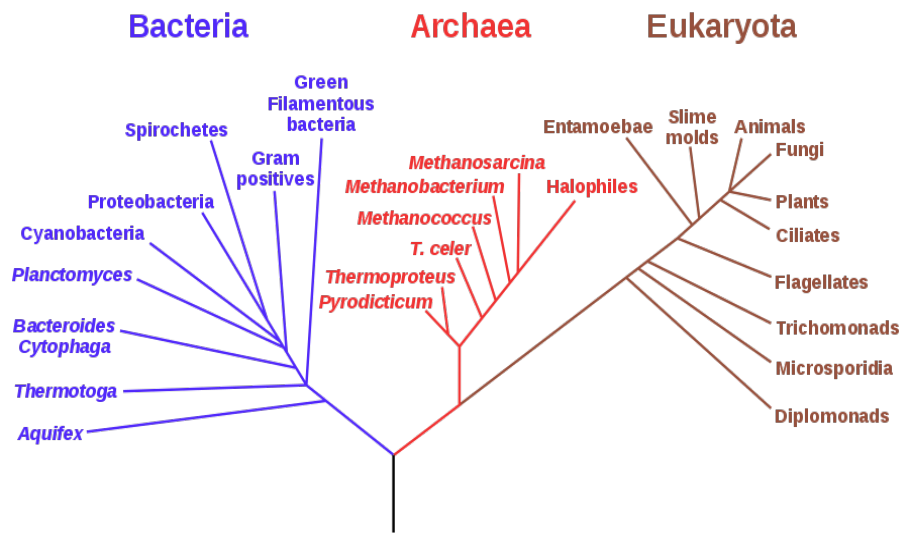


Figure 28:

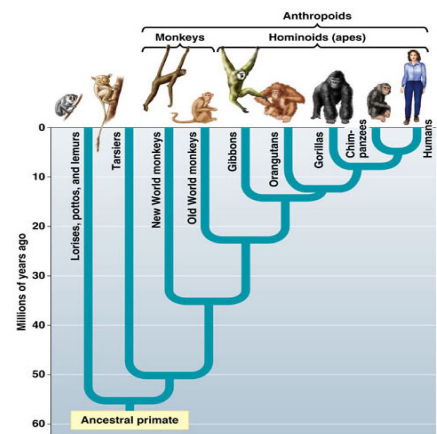


Figure 29:

directions.

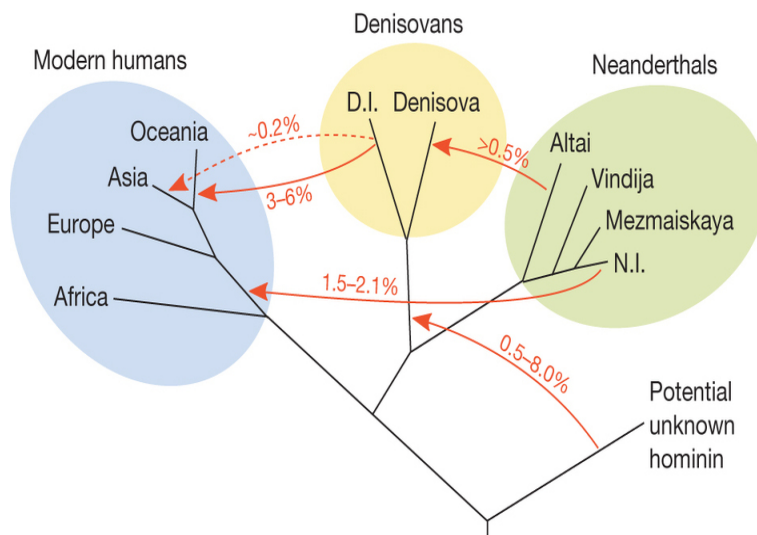
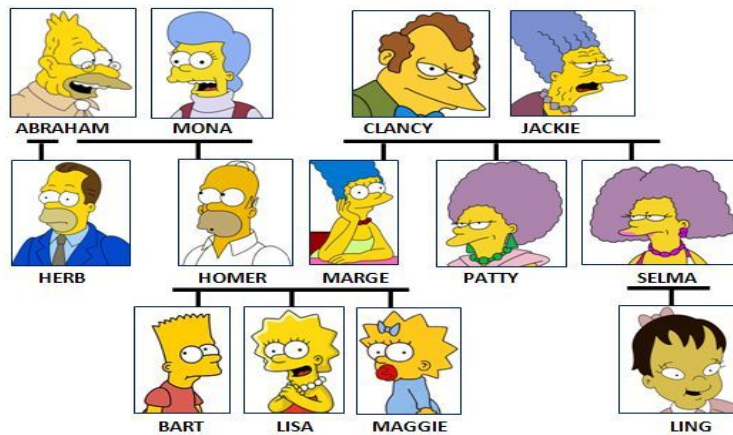
Besides, due to numerically unavoidable inbreeding, there are lots of cycles in this "tree" on relatively short time intervals, because, without, say 1000 years long cycles, you would have

$$2^{40} - \text{more than trillion grand-grand-grand-grand-...-grandmothers}$$

assuming maternal age of 25. This is greater than ¹⁰⁰⁰ the number of people who have ever lived on Earth.

And similar interbreeding/hybridisation linkages systematically violate/complement the phylogenetic pure tree structures, as it is clearly visible in the human genomes if you eye is aided with mathematics needed for this.

THE SIMPSONS



Most beautiful for a mathematician and most significant for a modern biologist are **molecular phylogenetic trees**, the nodes of which are marked by sequences of bases in DNA or amino acids in proteins.

Here again, the pure tree structure may be (often is) sprinkled with horizontal links, due to **viruses** and to **mobile genetic elements** which can move around within a genome, or that can be transferred from one species to another.

But there are also "perfect" trees in biology – binary cell lineage trees with exactly two immediate de-

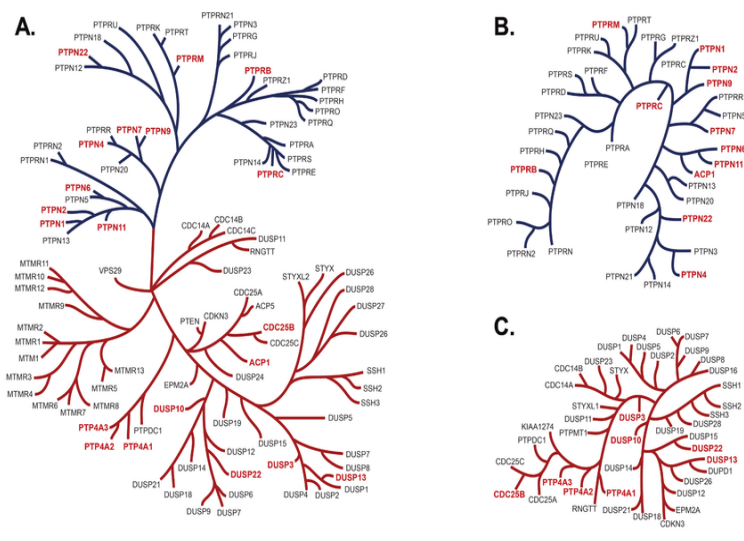


Figure 30: Dendrograms of evolution of protein tyrosine phosphatase and of its active sites

scendants for each nod which pictorially describe the history of cell colonies arising by cell division.
 (Even here "perfect treeness" is disrupted/complemented, e.g. by exchange of genetic material in unicellular organisms, such as bacterial conjugation and mating of yeast.)

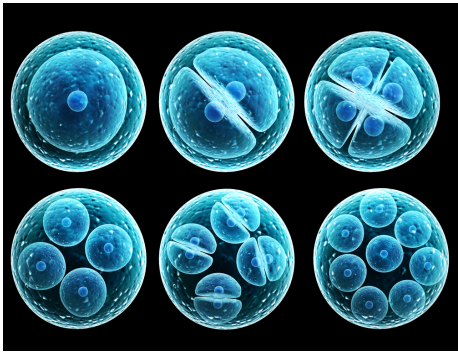


Figure 31: First three rounds of embryonic cell division

You whole body – a colony of 30-40 trillions cells, or rather the history of its development and growth, can be described by such a tree rooted in a single cell, called *zygote*, where cells in different parts of the body can be traced back to the zygote by different numbers of steps of cell divisions, which is regulated by sill

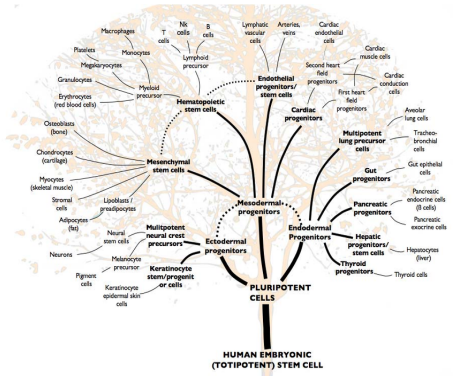


Figure 32: Cellular Differentiation Tree

elusive control mechanisms of the numbers of cells constituting particular types of tissues in the body.

Less obviously, trees make the main ingredients in organisation of the **energy landscape** levels of (large) molecules, e.g. (folded and unfolded) proteins, and in the **fitness landscapes** of evolution.

Here beware: the word "landscape" misdirects you imagination: what is commonly seen as "landscape" has rather primitive tree structure in it – mountains don't branch much.



It takes a bit of mathematical thinking to see trees in the multidimensional **energy** and/or **fitness landscapes** and to appreciate the significance of this tree-ness.

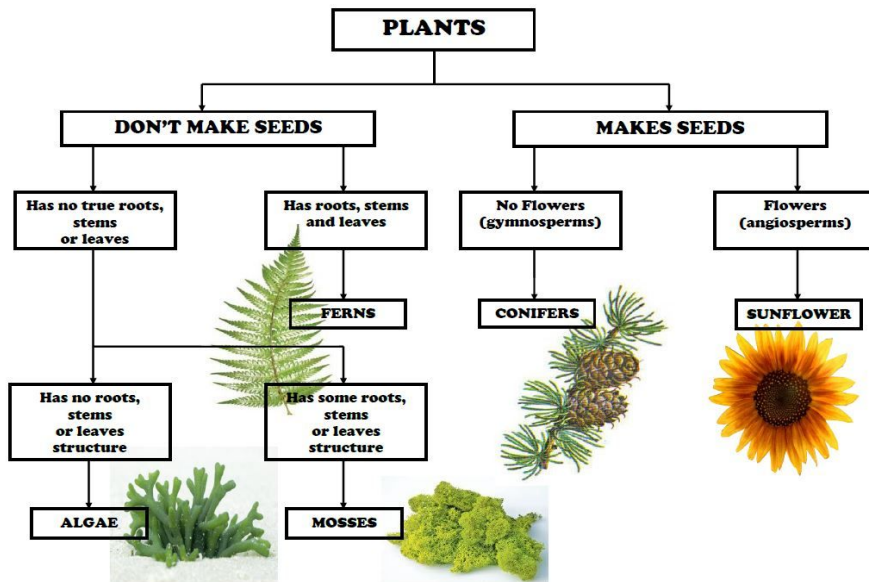
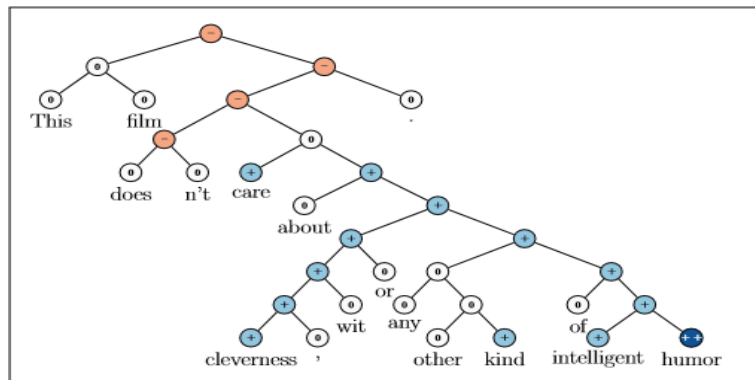
Also, tree-like patterns are persistent in products of the human mind.

Thus, a mathematical grammarian assigns **parse tree** structures to nested arrangements of phrases in sentences in the human languages, while the semantic meaning of a word or of a short phrase is reflected in the branching pattern of the tree of longer phrases containing it.

And the tree organisation of itemised ideas is ubiquitous in all kind of classification schemes, both within and without the individual human mind, be these ideas about objects, e.g. plants or about moves in a chess game.

Probably, all of what is going on in the human mind can be depicted as a conglomeration of several overlapping trees.

////////////////////////////////////v////////////////////////////////////
ADD



resolvable specificity of trees versus universality of
 general networks
 definition of sublevel trees

