

## Probability/Topology – Synopsis of lecture 1

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### What is probability?

We define the art of conjecture, or stochastic art, as the art of evaluating as exactly as possible the probabilities of things...

Jacob Bernoulli

**1713 The Law of Large Numbers** - One of 10-15 main focal Point in Mathematics stated by Gerolamo Cardano in **?1564?**).

Wrong Question!

Let us ask instead

What are mathematical principles from which probability theory **radiates?**

## 1653 Pascal's triangle of $C(n,k)$

"*Traite du triangle arithmetique avec quelques autres*"

1527 The  $TC(n,k)$ -triangle was published by Petrus Apianus.

1321 The numbers  $C(n,k)$  were computed by Levi Ben Gershon.

1048-1131 Omar Kayam claimed to know  $C(n,k)$  for expansion of  $(\mathbf{a} + \mathbf{b})^n$  and extracting  $n$ -th roots.

(1010-1070) In China, Yang Hui in his 1261 paper "Xiangjie Jiuzhang Suanfa" attributes  $C(n,k)$  to Jai Xian's (1010-1070) paper "Rújī Shìsuǒ":

*algorithms to extract  $n$ -th roots using binomial expansion of  $(\mathbf{a} + \mathbf{b})^n$*

(200 BC) The algorithm to construct the  $(\mathbf{a} + \mathbf{b})^n$  arithmetic triangle  $C(n,k)$  by the recursion

$$C(n,k) = C(n-1,k-1) + C(n-1,k):$$

Commentary to "Mrtasa njivani" written by Halayudha, in 10th century AD:

*linguistic analysis* of a sentence in "Chandahśastra" by Acharya Pingala ( circa 200 BC).

(In the modern terms Pingala, worked on the *coding theory*.)

**Reference.** *Localization in geometry and physics* by V. Pestun.

**1893 Poincaré Duality.** Another focal points in mathematics stated by Henri Poincaré in 1893 and expanded in his 1895 paper *Analysis Situs*.

(**PD** is an involution between invariants of  $k$  and  $(n - k)$  dimensional objects in  $n$ -dimensional spaces, which generalizes the orthogonal complementation to subspaces in the

## Hilbert/Euclidean/Pythagorus spaces.)

*Reference.* *Manifolds: Where Do We Come From? What Are We? Where Are We Going*, M.G.

**1982 Kesten's  $1/2$  theorem in 2D Percolation** (*Flory–Stockmayer 1941*) **Theory.** *For the square lattice in two dimensions the critical probability for bond percolation is  $1/2$ : the "signature" of the "stochastic fixed point" of the Poincare involution.*

(Flory–Stockmayer theory is about the cross-linking and gelation of step-growth polymers.)

*Random Linearized Topology, Random Topology Random Spectra, Expanders,*

**Reference.** Singularities, expanders and topology of maps. MG.

*Stochastic Linearization of the rules of inheritance*

**Reference.** Mendelian Dynamics and Sturtevant's Paradigm. MG

*Statistical Mechanics and Boltzmann-Shannon Entropy*

*Linearized Shannon Inequalities, Isoperimetric inequalities and Von Neumann Entropy*

**Reference** In a Search for a Structure, Part 1: On Entropy. MG

*Guth Duality between Homology Valued Probabilities of Multi-Particle Systems and the (non-linear) spectrum of the volume functionals.*

**Reference.** Morse Spectra, Homology Measures, Spaces of Cycles

*and Parametric Packing Problems.MG*

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*Graph valued Probabilities in Biology and Linguistic.*

*Stochastic Multi-Particle Representation of non-Linear Geometric PDE motivated by the biophysics of micelles.*