The range of a rotor walk

Laura Florescu, NYU
Joint work with Lionel Levine and Yuval Peres.
NYU

Rotor Router Walk

Definition

Let $G = (V, E)$ graph.

- Deterministic analogue of random walk
- Attach arrows (rotors) at each site pointing in any direction. At each step, rotate the arrow counter-clockwise by 90 degrees and then move the particle in that direction.
- In the square grid $\mathbb{Z}^2$, successive exits could repeatedly cycle through the sequence North, East, South, West.

Graphs

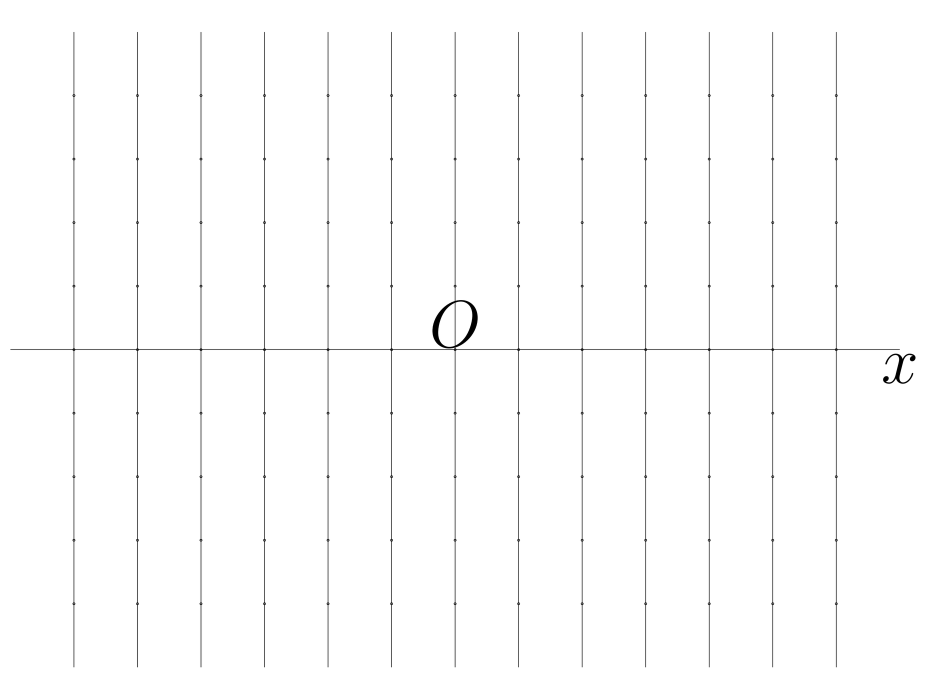
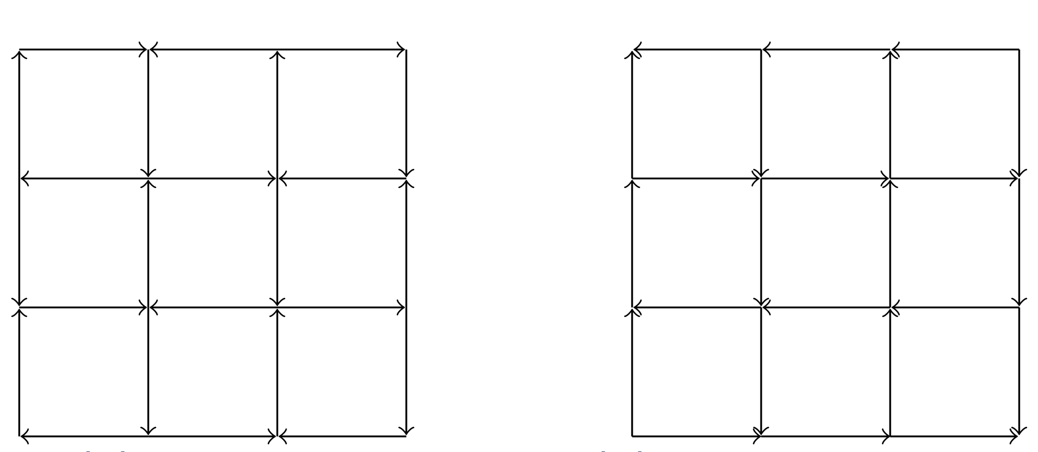


Figure: Comb graph



(a) F-Lattice (b) Manhattan lattice

Figure: Two different periodic orientations of the square grid with indegree and outdegree 2.

$\#R_t$: number of distinct sites visited by time t

Range on $\mathbb{Z}^d$

Theorem

For any Eulerian graph G of bounded degree satisfying a volume growth condition, the number of distinct sites visited by a rotor walk started at o in t steps satisfies

$$\#R_t \geq ct^{d/(d+1)}.$$

for a constant $c > 0$ depending only on G (and not on ρ or m).

Range on comb graph

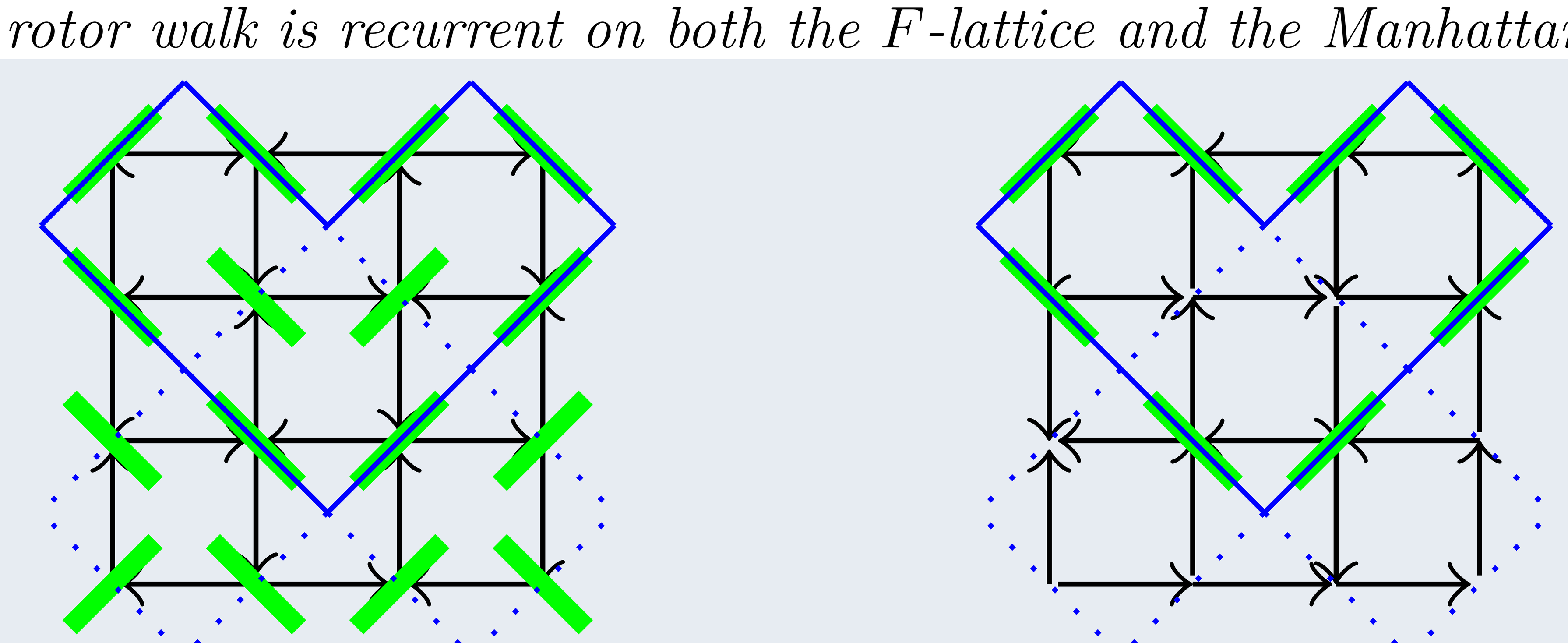
Theorem

For uniform rotor walk on the comb graph, $\#R_t$ has order $t^{2/3}$ and the asymptotic shape of R_t is a diamond.

Recurrence of Manhattan and F-lattices

Theorem

Uniform rotor walk is recurrent on both the F-lattice and the Manhattan lattice.



(a) F-Lattice (b) Manhattan lattice

Figure: Percolation on $\mathbb{L}$: dotted blue edges are open, solid blue edges are closed. Shown in green are the corresponding mirrors on the F-lattice (left) and Manhattan lattice.

Simulations

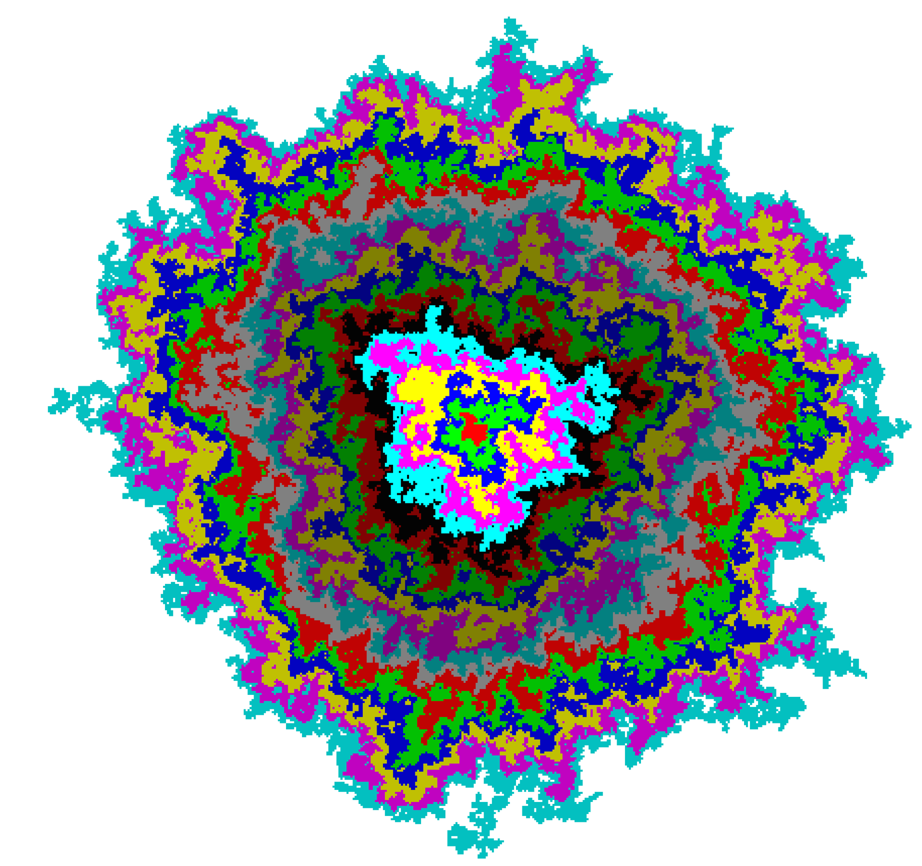


Figure: The range on $\mathbb{Z}^2$.Colors: excursions.

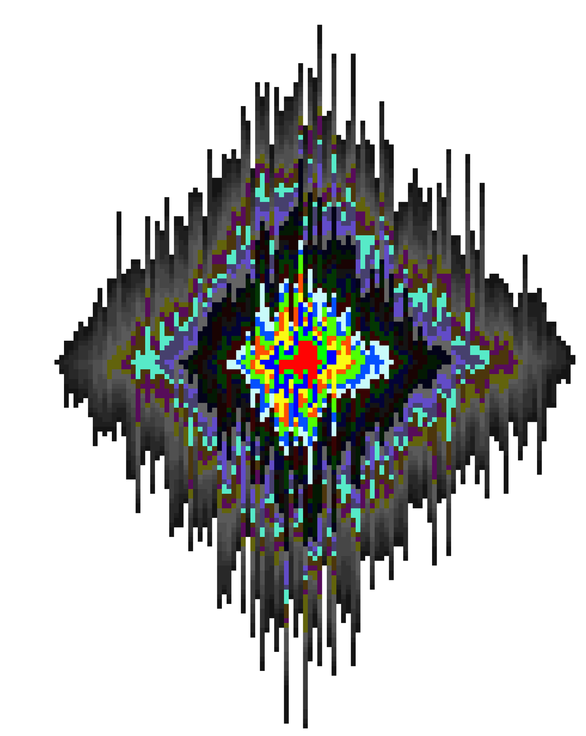


Figure: Set of sites visited on comb graph

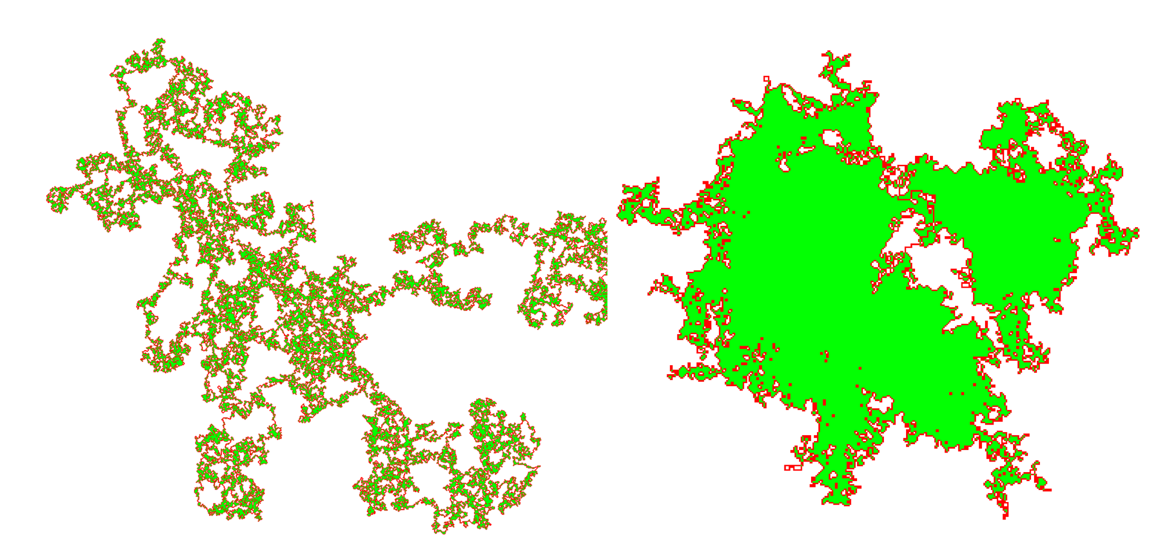


Figure: F-lattice and the Manhattan lattice.

References

[1] Florescu, Levine, Peres, *The range of a rotor walk*, 2014, arxiv: 1408.5533.