Partial Differential Equations, Spring 2018
Homework VII: Numerical Methods for PDEs

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The maximum number of points is 40 (excluding extra credit).

Submit the solution to this homework via NYU Classes only. **You must submit a short (say 2 page) writeup** (in PDF format) documenting what you did, that is, an explanation of what the code does and answers to the questions posed below. Make sure to include figures of your results in the writeup (Hint: use EPS/PDF format for best quality, or, if you must, PNG and not JPG format). If you have never tried LaTex and wish to do so, I suggest you try to install LyX, a graphical WYSYWG interface to LaTex. **Also attach your MATLAB codes to the homework**. Note: Everyone's answers (plots, code and especially discussion of results) **must be unique and their own** – any identical answers will be considered cheating and penalized accordingly!

You can start from the MATLAB code HeatNumerical.m that we covered in class and can be downloaded from the course webpage.

1. [20 pts] Solve the heat equation on the interval $0 < x < \pi$,

$$\begin{aligned} u_t &= u_{xx} \\ u(0,t) &= 0 \\ u(\pi,t) &= 0 \\ u(x,0) &= \exp\left(-\left(x-\pi/2\right)^2/0.01\right) \end{aligned}$$

   numerically and plot the solution at several points in time $t > 0$ (choose those points in time smartly). Make sure you use different symbols/colors/line styles for the different t's at which you plot your solution, and make sure to put a legend on the figure and label your axes.
2. [20pts] Now let's consider the same problem but on the whole real line,

$$\begin{aligned} u_t &= u_{xx} \\ u(x,0) &= \exp\left(-\left(x-\pi/2\right)^2/0.01\right). \end{aligned}$$

   (a) [10 pts] Solve this problem analytically by hand. Compare your analytical solution to the the solution from Problem #1 for several values of t. What do you observe? How large does t need to be for you to be able to see the difference on your plots?
   (b) [5 pts] Now plot the difference between the two solutions at several values of t and discuss what you observe.
   (c) [5 pts] Do you think you can trust your numerical results to be correct, and why do you think that? Hint: Another way to ask this is to quantify *how accurate* your answers are.
3. [Extra Credit challenge, up to 10pts, 5pts per BC] Solve Problem #1 with homogeneous Neumann boundary conditions instead of homogeneous Dirichlet. Do the same for periodic "boundary conditions." Explain how the BC changes the solution based on your numerical observations/plots.