Partial Differential Equations, Spring 2020
Homework IV: The Wave and Diffusion Equations

Aleksandar Donev
Courant Institute, NYU, donev@courant.nyu.edu

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Total number of points is 65.

1. (7.5pts) Solve the wave equation $u_{tt} = c^2 u_{xx}$ on the whole real line, with the initial condition $u(x,0) = e^{-2|x|}$ and $u_t(x,0) = \sin x$.
2. (12.5 pts) Use a change of variables/coordinates $(x,y) \longleftrightarrow (s,t)$ to determine the general solution (7.5pts) to the PDE
$$u_{xx} - 4u_{yy} = x - 2y. \tag{1}$$
Can you find a specific solution (5pts) in the whole xy plane that satisfies the boundary conditions $u(x,0) = x^2$ and $u(0,y) = y^2$?
[*Hint: Recall it is a good idea to check the solution at the end.*]
3. (10pts) Solve the diffusion equation $u_t = u_{xx}$ on the whole real line, with the initial condition
$$u(x,0) = \exp\left(-|x|\right) \tag{2}$$
4. (10pts) Solve the PDE on the whole real line ($x \in \mathbb{R}$, $t > 0$)
$$u_t - u_{xx} = \begin{cases} \exp\left(-x\right) & \text{for } x > 0 \\ 0 & \text{for } x \le 0. \end{cases}$$
with the initial condition (2).
5. (10pts) Use Duhamel's principle to solve the PDE (1) with the boundary conditions
$$u(x=0,y) = 0$$
$$u_x(x=0,y) = 0.$$
[*Hint: The formula for the answer is in the lecture notes on Duhamel's principle.*]
6. (15pts) Use Duhamel's principle to solve the forced advection equation on the whole real line,
$$u_t + c\,u_x = f(x,t),$$
with an initial condition $u(x,0) = 0$. Write down all steps of the derivation (10pts).
(5pts): Follow the method of characteristics to solve the PDE (and compare the answers, of course).