Model Reduction and Mode Computation for Damped Resonant MEMS

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SIAM CSE 07, 19 Feb 2007

Collaborators

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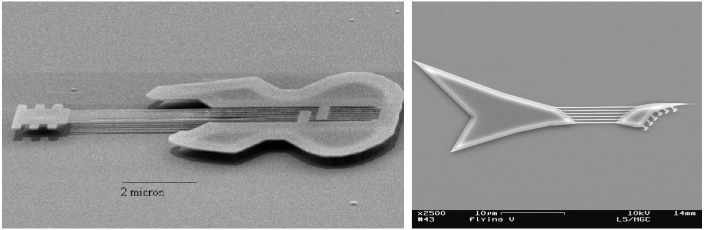
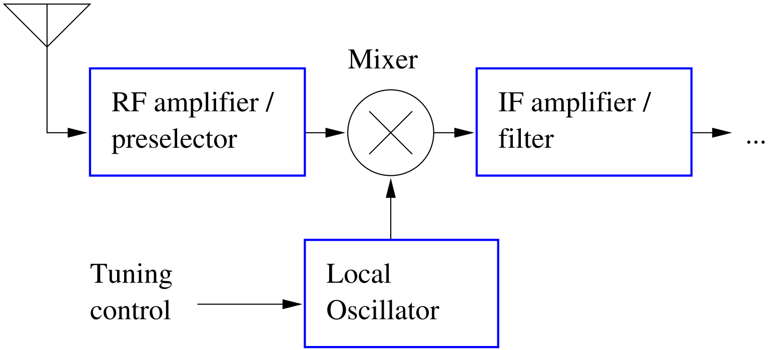
Resonant MEMS



Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Favorite application: radio on chip
- Close second: really high-pitch guitars

The Mechanical Cell Phone

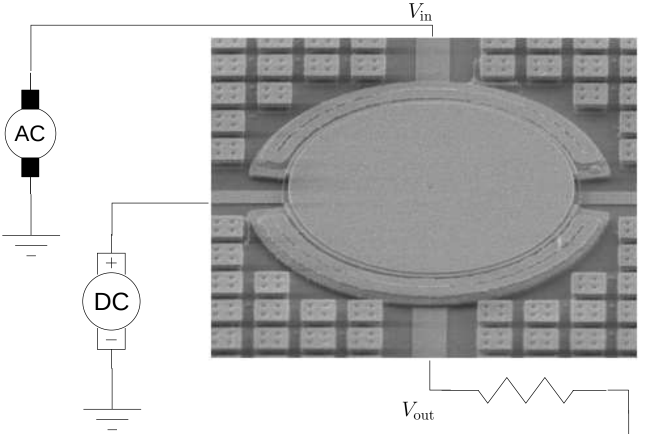


- Your cell phone has many moving parts!
- What if we replace them with integrated MEMS?

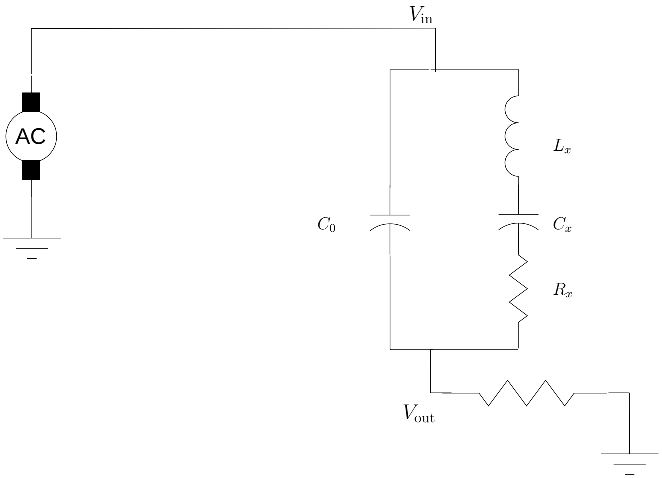
Ultimate Success

"Calling Dick Tracy!"

Example Resonant System



Example Resonant System



The Designer’s Dream

Ideally, would like

- Simple models for behavioral simulation
- Parameterized for design optimization
- Including all relevant physics
- With reasonably fast and accurate set-up

We aren’t there yet. Today, some progress on the last two.

Damping and Q

Designers want high *quality of resonance* (Q)

- Dimensionless damping in a one-dof system

$$\frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)$$

- For a resonant mode with frequency $\omega \in \mathbb{C}$:

$$Q := \frac{|\omega|}{2\,\mathrm{Im}(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}$$

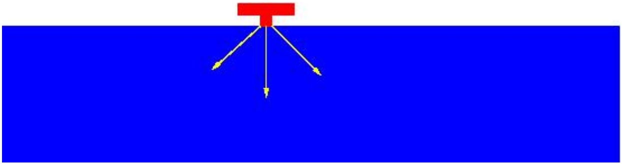
Damping Mechanisms

Possible loss mechanisms:

- Anchor loss
- Thermoelastic damping
- Other material losses
- Fluid damping

Our goal: Reduced models that include these effects.

Perfectly Matched Layers

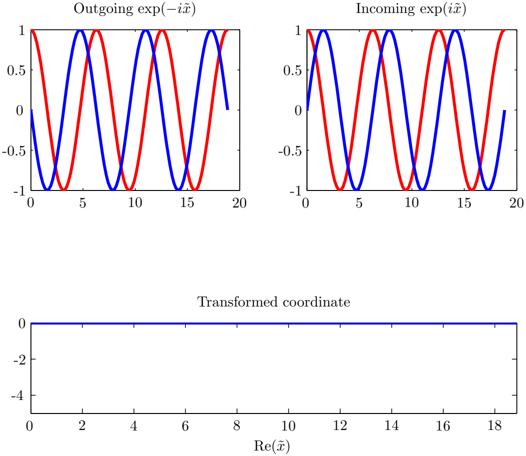


Model substrate as semi-infinite with a

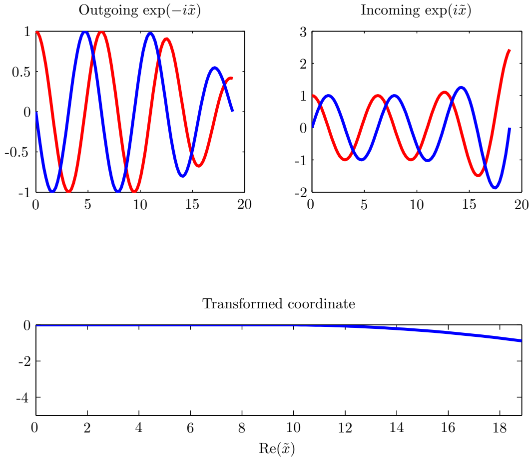
Perfectly Matched Layer (PML).

- Complex coordinate transformation
- Generates a "perfectly matched" absorbing layer
- Idea works with general linear wave equations

Perfectly Matched Layers by Picture



Perfectly Matched Layers by Picture



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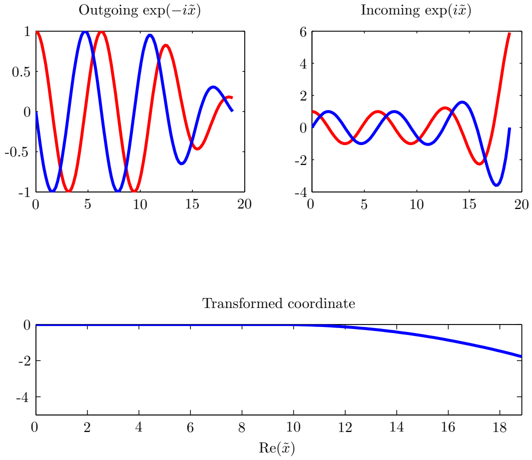

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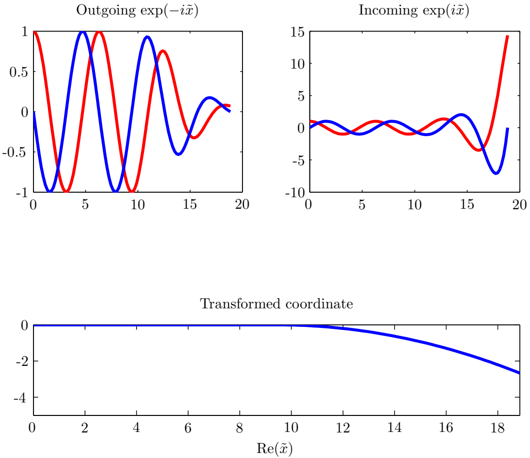

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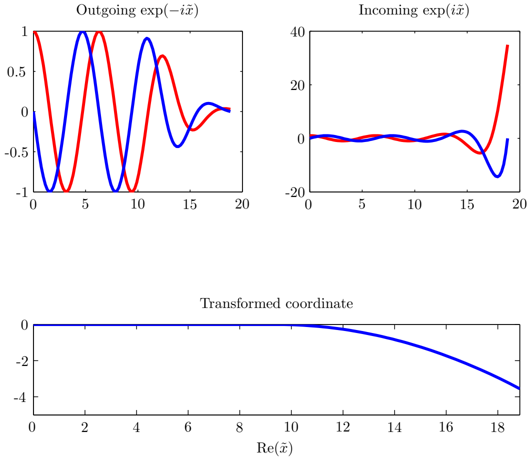

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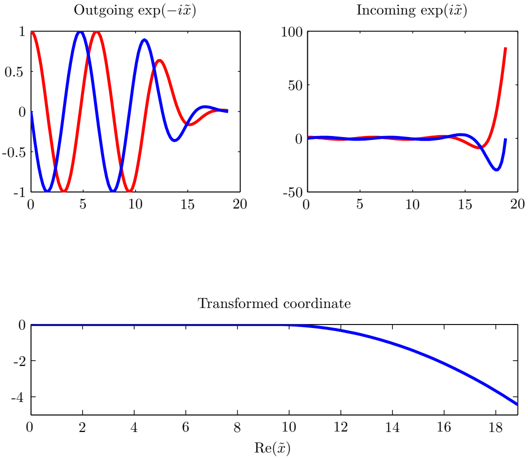
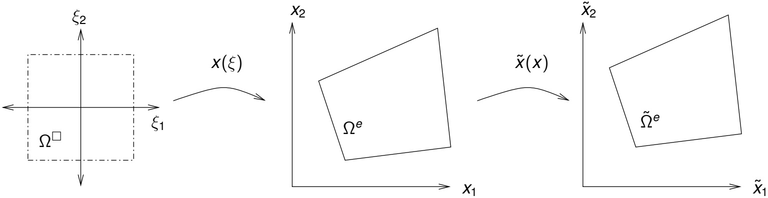


Finite Element Implementation



- Combine PML and isoparametric mappings

$$\mathbf{k}^e = \int_{\Omega^\square} \tilde{\mathbf{B}}^T \mathbf{D} \tilde{\mathbf{B}} \tilde{J} \, d\Omega^\square$$
$$\mathbf{m}^e = \int_{\Omega^\square} \rho \mathbf{N}^T \mathbf{N} \tilde{J} \, d\Omega^\square$$

- Matrices are *complex symmetric*

Complex Symmetry

Discretized (forced) problem + fixed PML take the form:

$$(K - \omega^2 M)u = f, \text{ where } K = K^T, M = M^T$$

Can still characterize u as a stationary point of

$$I(u) = \frac{1}{2}u^T(K - \omega^2 M)u - u^T f.$$

Eigenvalues of (K, M) are stationary points of

$$\rho(u) = \frac{u^T K u}{u^T M u}$$

First-order accurate vectors $\implies$
second-order accurate eigenvalues.

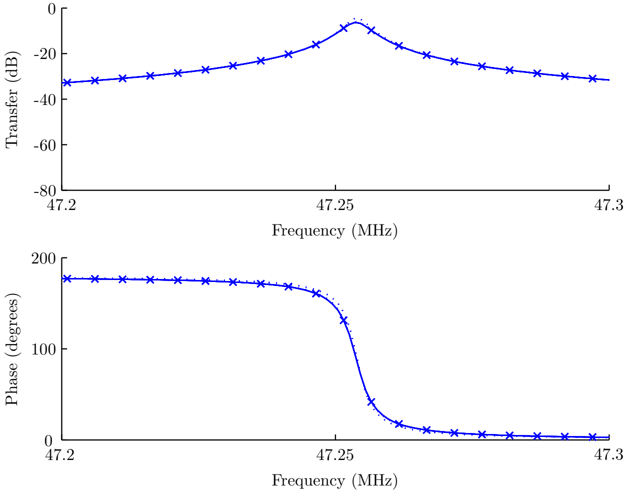
Accurate Model Reduction

- Usual: Orthogonal projection onto Arnoldi basis V.
- Us: Build new projection basis from V:

$$W = \text{orth}[\text{Re}(V), \text{Im}(V)]$$

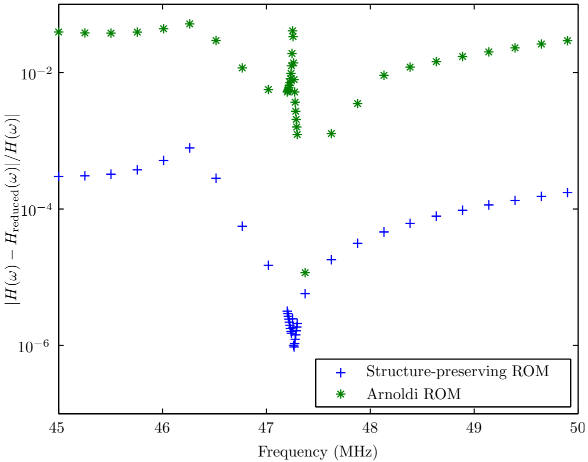
- span(W) contains both $\mathcal{K}_n$ and $\bar{\mathcal{K}}_n$
  $\Longrightarrow$ double digits correct vs. projection with V
- W is a real-valued basis
  $\Longrightarrow$ projected system is complex symmetric

Model Reduction Accuracy



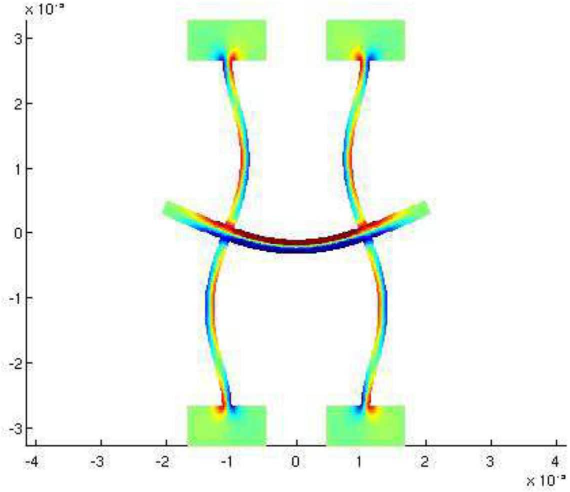
Results from ROM (solid and dotted lines) near indistinguishable from full model (crosses)

Model Reduction Accuracy



Preserve structure $\implies$
get twice the correct digits

Thermoelastic Damping (TED)



Thermoelastic Damping (TED)

u is displacement and $T = T_0 + \theta$ is temperature

$$
\begin{aligned}
\sigma &= C\epsilon - \beta\theta 1 \\
\rho\ddot{u} &= \nabla \cdot \sigma \\
\rho c_v \dot{\theta} &= \nabla \cdot (\kappa \nabla \theta) - \beta T_0 \operatorname{tr}(\dot{\epsilon})
\end{aligned}
$$

- Coupling between temperature and volumetric strain:
  - Compression and expansion $\implies$ heating and cooling
  - Heat diffusion $\implies$ mechanical damping
  - Not often an important factor at the macro scale
  - Recognized source of damping in microresonators
- Zener: semi-analytical approximation for TED in beams
- We consider the fully coupled system

Nondimensionalized Equations

Continuum equations:

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ \ddot{u} &= \nabla \cdot \sigma \\ \dot{\theta} &= \eta\nabla^2\theta - \mathrm{tr}(\dot{\epsilon}) \end{aligned}$$

Discrete equations:

$$\begin{aligned} M_{uu}\ddot{u} + K_{uu}u &= \xi K_{u\theta}\theta + f \\ C_{\theta\theta}\ddot{\theta} + \eta K_{\theta\theta}\theta &= -C_{\theta u}\dot{u} \end{aligned}$$

- Micron-scale poly-Si devices: ξ and η are $\sim 10^{-4}$.
- Linearize about $\xi = 0$

Perturbative Mode Calculation

Discretized mode equation:

$$\begin{aligned}(-\omega^2 M_{uu} + K_{uu})u &= \xi K_{u\theta}\theta \\ (i\omega C_{\theta\theta} + \eta K_{\theta\theta})\theta &= -i\omega C_{\theta u} u\end{aligned}$$

First approximation about $\xi = 0$:

$$\begin{aligned}(-\omega_0^2 M_{uu} + K_{uu})u_0 &= 0 \\ (i\omega_0 C_{\theta\theta} + \eta K_{\theta\theta})\theta_0 &= -i\omega_0 C_{\theta u} u_0\end{aligned}$$

First-order correction in ξ:

$$-\delta(\omega^2) M_{uu} u_0 + (-\omega_0^2 M_{uu} + K_{uu})\delta u = \xi K_{u\theta}\theta_0$$

Multiply by u_0^T:

$$\delta(\omega^2) = -\xi \left(\frac{u_0^T K_{u\theta}\theta_0}{u_0^T M_{uu} u_0} \right)$$

Zener's Model

1. Clarence Zener investigated TED in late 30s-early 40s.
2. Model for beams common in MEMS literature.
3. “Method of orthogonal thermodynamic potentials” == perturbation method + a variational method.

Comparison to Zener's Model

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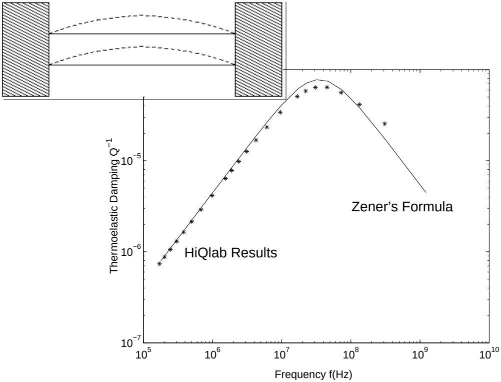

- Comparison of fully coupled simulation to Zener approximation over a range of frequencies
- Real and imaginary parts after first-order correction agree to about three digits with Arnoldi

General Picture

If $w^* A = 0$ and $Av = 0$ then

$$\delta(w^* A v) = w^* (\delta A) v$$

This implies

- If $A = A(\lambda)$ and $w = w(v)$, have

$$w^*(v) A(\rho(v)) v = 0.$$

  ρ stationary when $(\rho(v), v)$ is a nonlinear eigenpair.
- If $A(\lambda, \xi)$ and w_0^* and v_0 are null vectors for $A(\lambda_0, \xi_0)$,

$$w_0^* (A_\lambda \delta\lambda + A_\xi \delta\xi) v_0 = 0.$$

Conclusions

- Resonant MEMS have lots of interesting applications
- Designers want reduced models with relevant physics
- Damping is crucial, but not well handled in general
- Our work: use equation structure in making reduced models with damping (modal or more general)