Mathematical modeling of a viscida network

Christiana Mavroyiakoumou

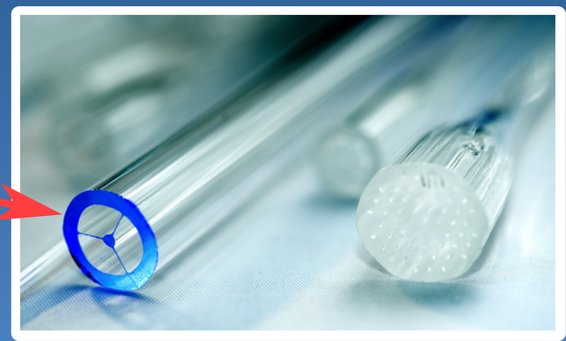
Department of Mathematics, University of Michigan

Supervisors: Dr I.M.Griffiths, Prof. P.D.Howell (University of Oxford)

Introduction

Background Information:

Microstructured optical fibers (MOFs), are gaining increasing interest for their use in broadband communications, sensing, and medicine. Unlike conventional fibers, MOFs guide light using a geometrical array of air holes along their length. These holes provide an effective difference in the refractive index, and so light passing through the core is guided through total internal reflection.

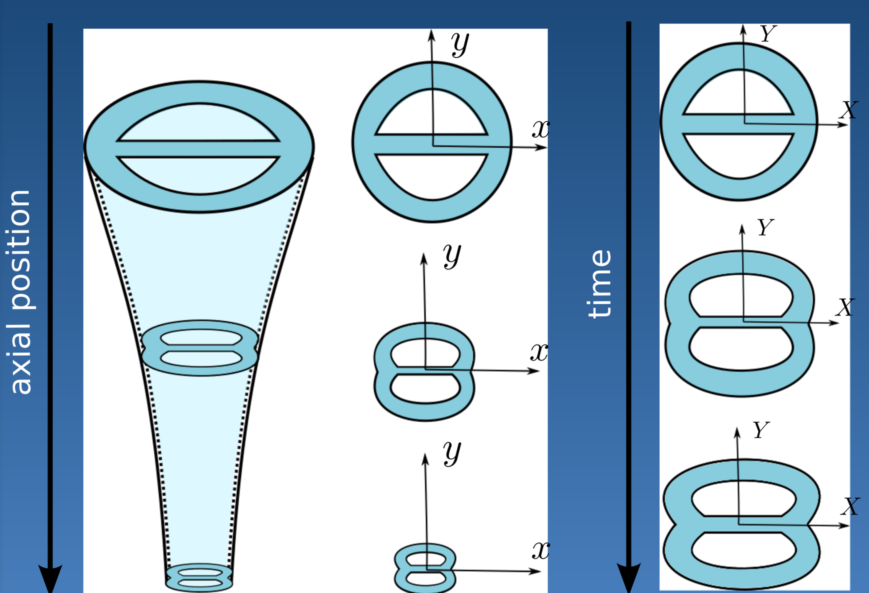


Motivation & Aim

This project aims to predict undesired shape deformations.

The motivation of this project is primarily an interest in developing a general model to describe a network of interconnected viscous sheets (viscidas) which evolve under the action of surface tension.

2D model for the shape evolution of the cross-section of a MOF



We exploit various geometrical features to simplify the problem. The fiber cross-section is slowly varying in the axial direction, so in a suitable Lagrangian frame of reference, the shape evolution of the cross-section is reduced to a classical 2D Stokes-flow-free-boundary problem. This is coupled with a 1D axial-stretching problem, determining the size of the cross-section as a function of axial distance.

Evolution equation

$$\frac{\partial}{\partial \tau}\left[\left(1+\frac{\tau}{2}\right)\frac{\partial^2 \theta_i}{\partial \xi_i^2}\right] = A_i(\tau)\sin\theta_i + B_i(\tau)\cos\theta_i$$

$$i \in \{1,2,3\}$$

Boundary conditions

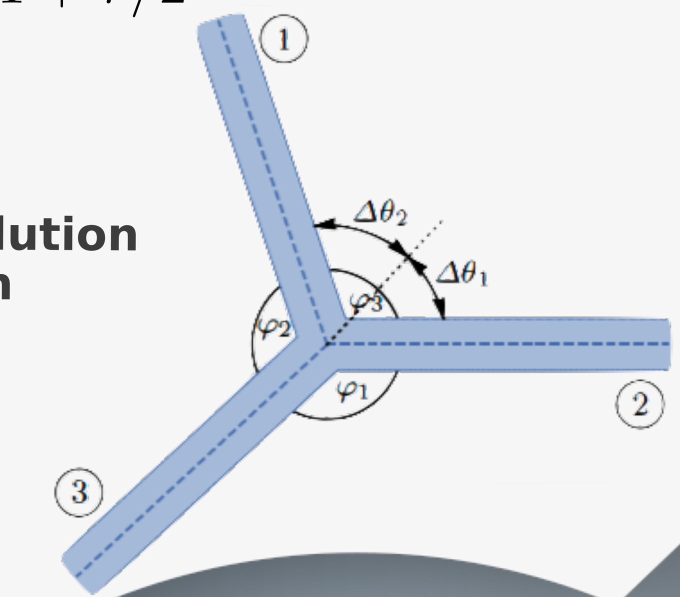
1. Angle evolution at corners

$$\alpha_i(\tau) = \frac{2\pi}{3} + \frac{\alpha_i(0) - 2\pi/3}{1+\tau/2} = (-1)^{i+1}[\theta_i(0,\tau) - \theta_3(0,\tau)]$$

$$\beta_i(\tau) = \frac{2\pi}{3} + \frac{\beta_i(0) - 2\pi/3}{1+\tau/2} = (-1)^{i+1}[\theta_i(\ell_3,\tau) - \theta_3(\ell_i,\tau)]$$

$$i \in \{1,2,3\}$$

Angle evolution at junction



2. Ends must meet at the junctions

$$\int_0^{\ell_1}\begin{bmatrix}\cos\theta_1\\ \sin\theta_1\end{bmatrix}\mathrm{d}\xi_1 = \int_0^{\ell_2}\begin{bmatrix}\cos\theta_2\\ \sin\theta_2\end{bmatrix}\mathrm{d}\xi_2 = \int_0^{\ell_3}\begin{bmatrix}\cos\theta_3\\ \sin\theta_3\end{bmatrix}\mathrm{d}\xi_3$$

3. Zero net bending moment at junction

$$\frac{\partial}{\partial \tau}\left[\left(1+\frac{\tau}{2}\right)\left(\frac{\partial\theta_1}{\partial\xi_1}+\frac{\partial\theta_2}{\partial\xi_2}+\frac{\partial\theta_3}{\partial\xi_3}\right)\right]=0$$

at $\xi_i = 0, \xi_i = \ell_i$ for $i \in \{1,2,3\}$

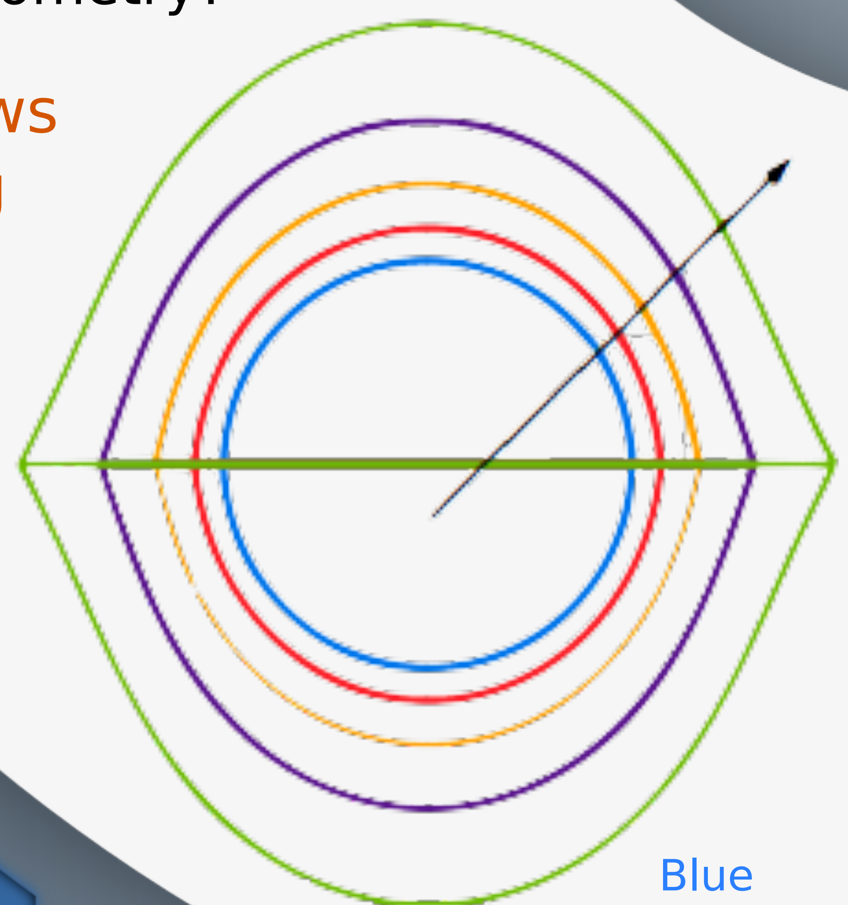
4. Zero resultant forces

$$\sum_{i=1}^{3} A_i(\tau) = 0$$

$$\sum_{i=1}^{3} B_i(\tau) = 0$$

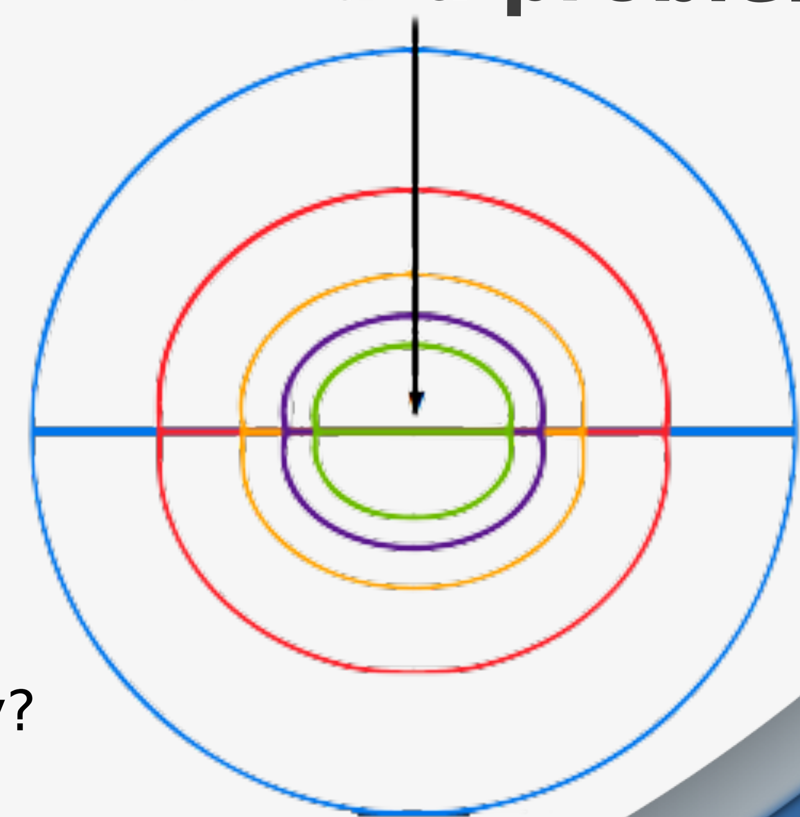
Inverse problem

What preform will produce the desired MOF geometry?



Forward problem

How will the final MOF look like, for a given preform geometry?



Governing equations

force and moment balance

$$\frac{\partial M}{\partial s} = A(t)\sin\theta - B(t)\cos\theta$$

$$M = \frac{h^3}{3}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial s}\right)$$

thickness evolution

$$\frac{Dh}{Dt} = \frac{1}{2}$$

centerline position

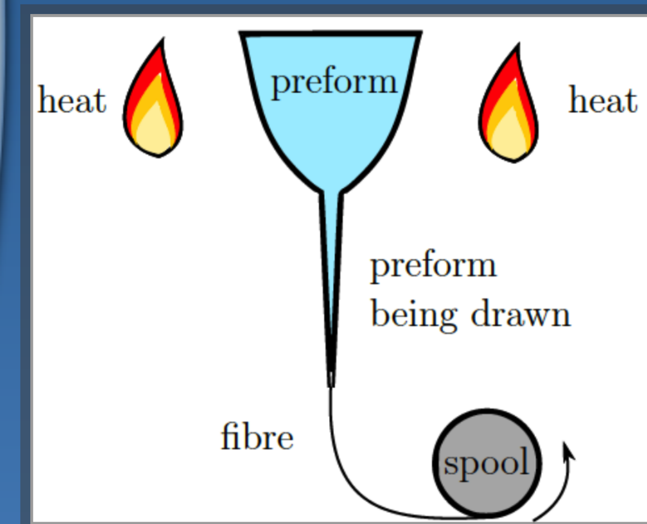
$$\frac{\partial x}{\partial s} = \cos\theta$$

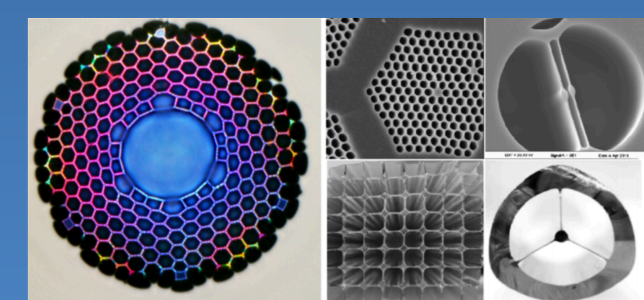
$$\frac{\partial y}{\partial s} = \sin\theta$$

We require one initial condition and four boundary conditions, which are usually specified by:

1. Inclination angle θ at each end
2. Position of each end of the viscida
3. Bending moment M applied at each end
4. Tensions A and B.

How do you actually make them?





Impact of model and applications

1. Inverse problem is easier to solve.
2. No need for expensive computational methods.
3. We get useful insights that are not available through experimental work.
4. We reduce the cost of trial and error, saving a lot of money.

References

The poster design was adapted from the design of Felix Breuer http://blog.felixbreuer.net/2010/10/24/poster.html

[1] Mavroyiakoumou, C., Griffiths, I. M., & Howell, P. D. (2017). MSc thesis.

[2] Griffiths, I. M., & Howell, P. D. (2007). Journal of Fluid Mechanics, 593, 181-208.