Modeling flying formations as flow-mediated matter

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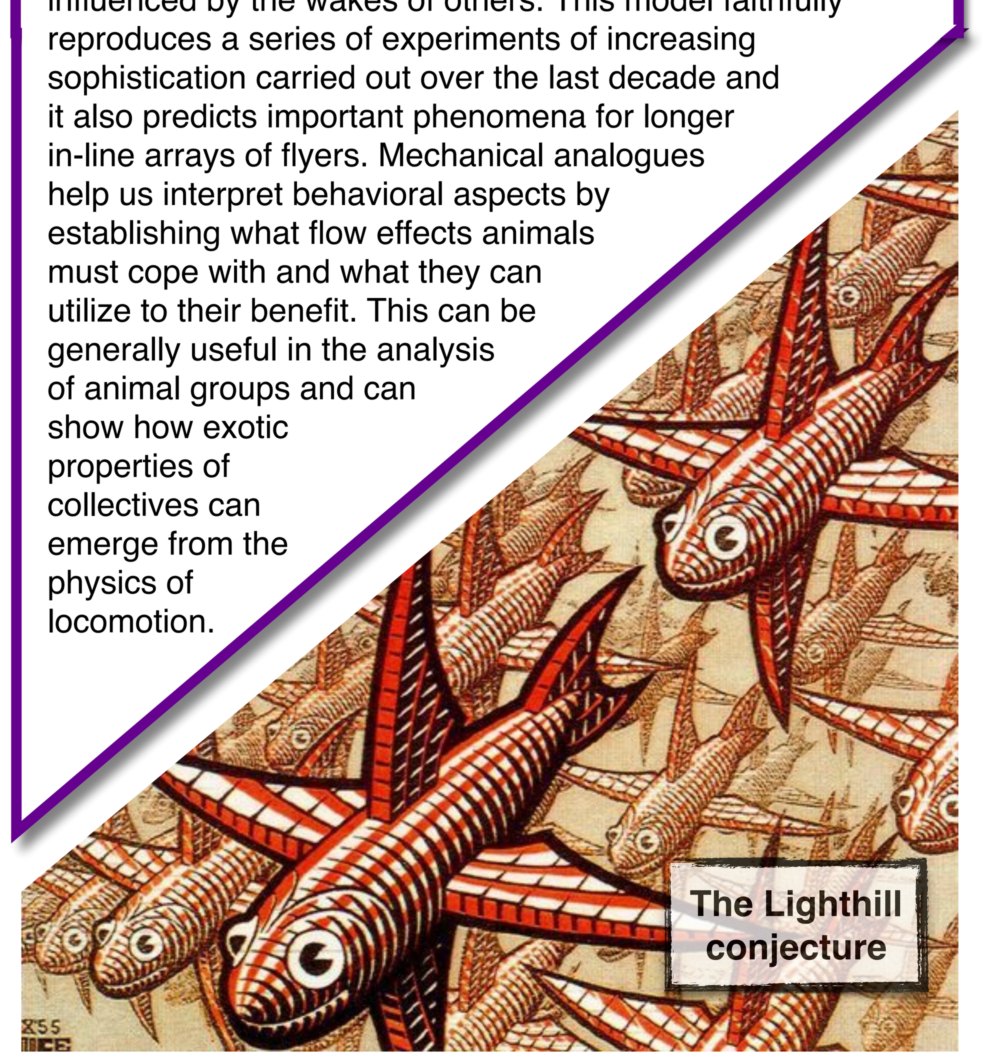
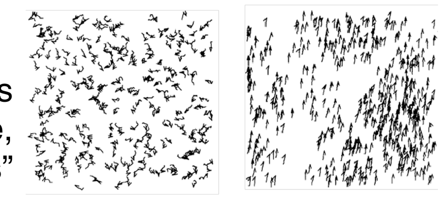
Motivation

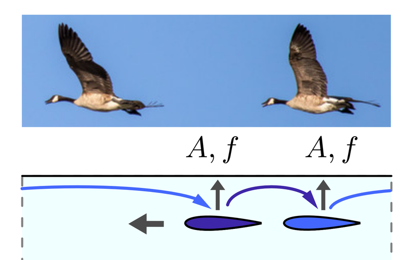
Collective locomotion of swimming and flying animals is fascinating in terms of individual-level fluid mechanics and group-level structure and dynamics. We bridge and relate these scales through a model of formation flight that views the collective as a material whose properties arise from flow-mediated interactions among its members. We formulate an aerodynamic model that describes how flapping flyers produce wake signals and how the are influenced by the wakes of others. This model faithfully reproduces a series of experiments of increasing sophistication carried out over the last decade and it also predicts important phenomena for longer in-line arrays of flyers. Mechanical analogues help us interpret behavioral aspects by establishing what flow effects animals must cope with and what they can utilize to their benefit. This can be generally useful in the analysis of animal groups and can show how exotic properties of collectives can emerge from the physics of locomotion.

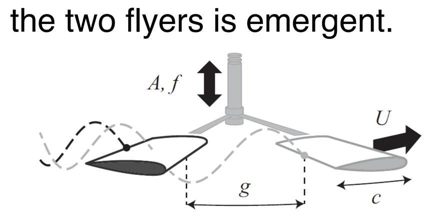


2 Emergent spacing

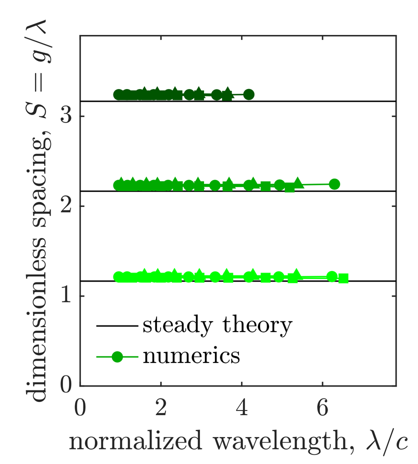


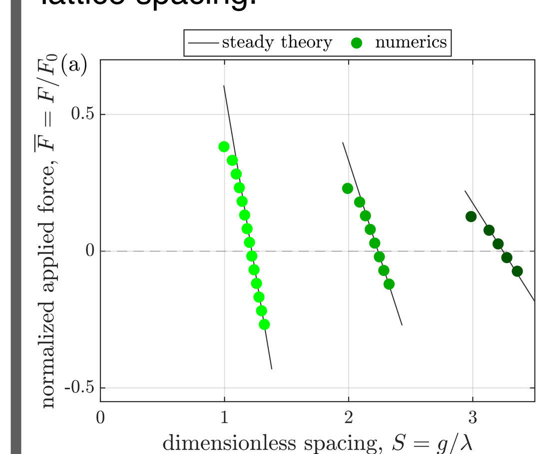
In Vicsek-type models each individual tries to align with its neighbors plus some noise, where this noise is essentially the "mistakes" a flyer can make in evaluating their direction of motion. When the noise is decreased, phase transitions from disordered to ordered states can be triggered. We use elements from this type of models in the second setup we consider, where now the spacing between the two flyers is emergent.





We fix the flapping kinematics (A and f) and find that the two flyers reach a stable configuration with a separation distance that is a near-integer multiple of the wavelength traced out by each flyer (U/f). This suggests an analogy with a crystal that has regularly ordered atoms with a lattice spacing.

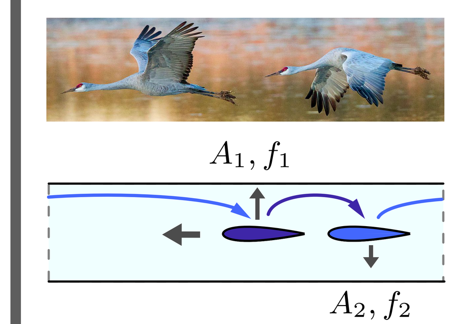


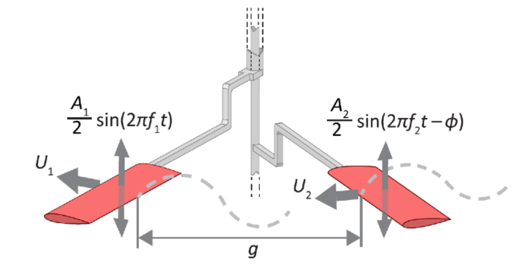


When a steady force is applied on the follower, multiple stable positions exist for the follower. The degree of stability depends on the strength of the hydrodynamic force that tends to restore the follower's position if perturbed. Therefore, the fluid force resembles a Hookean spring, with the spring constant associated to the slope of the force vs spacing plot.

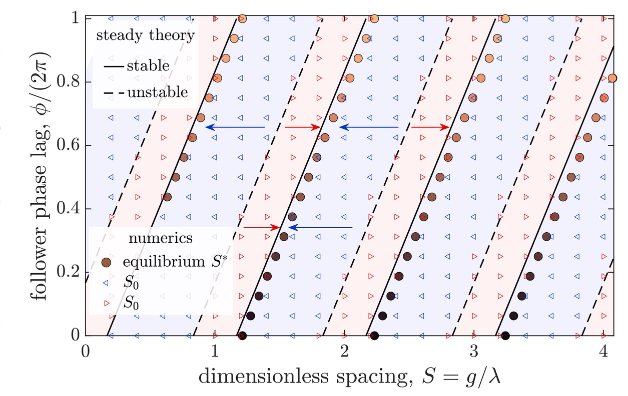
3 Kinematic individuality

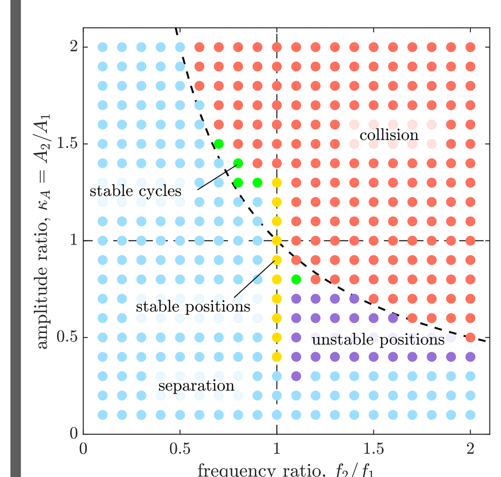
The system becomes more realistic when the two flyers are allowed to have synchronized or asynchronous flapping and dissimilar kinematics.





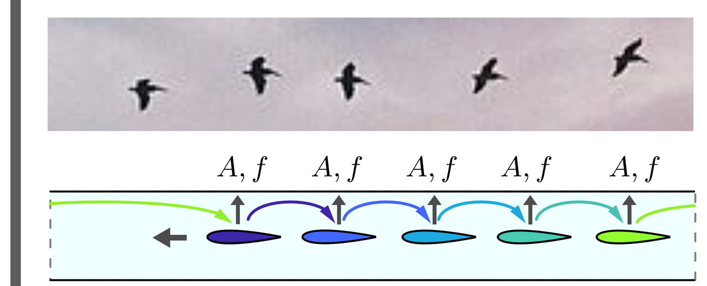
Our findings show that even when there is a phase lag between the flapping motions of the leader and the follower, the follower can still fall into specific positions behind the leader due to the wake interactions and the pair can move together cohesively.



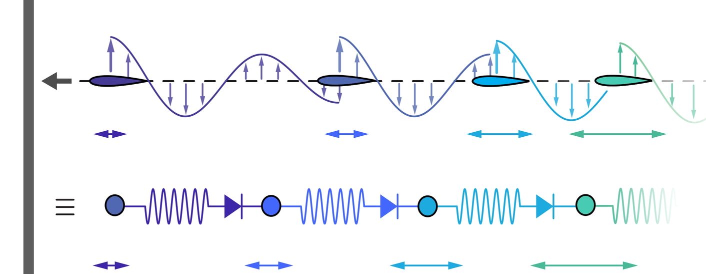


Mapping out the dynamics in the amplitude- vs frequency-ratio space, reveals that the follower falls into stable positions behind the leader even when it is underdriven. This occurs when $f_1 = f_2$. Stable cycles are also found in the vicinity of $f_1 = f_2$ when the follower is overdriven. The flow interactions act as a "bond" that maintains the cohesion of the pair even when the follower is only weakly flapping or when it is flapping faster than the leader.

4 Few-flyer system

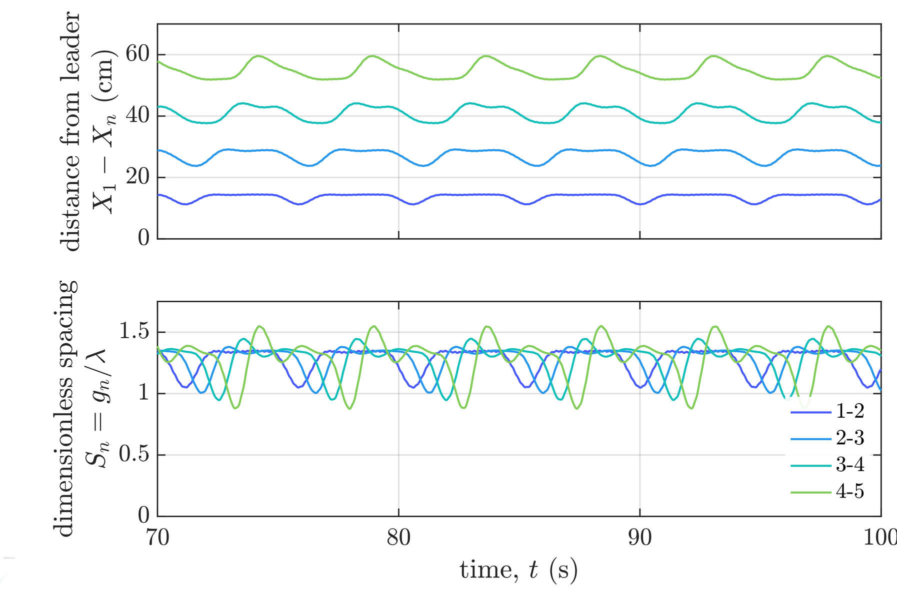


From systems with two flyers we now move to few flyers (up to five). Each individual emits a wake signal that influences its nearest downstream neighbor, which then erases it and replaces it with its own. This situation can be viewed as a system of a linear chain of masses linked to nearest neighbors by linear springs that are diodic and that have the same spring constant.

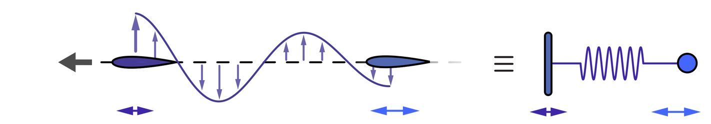


To understand the mechanism behind the amplified fluctuations we consider each pair of flyers as isolated from the rest of the group given the one-way interactions. Then the leader in a pair can be thought of playing the role of the driving source that forces the follower to oscillate.

We observe positional fluctuations that are correlated in time, propagate down the group like traveling waves, and grow in amplitude down the group. As collective excitations that propagate in a lattice, the emergent dynamics share some general features with conventional longitudinal displacement waves, e.g. phonons in atomic and molecular crystals. We call these positional fluctuations "flonons", for flow-mediated fluctuations among flapping, flocking flyers.

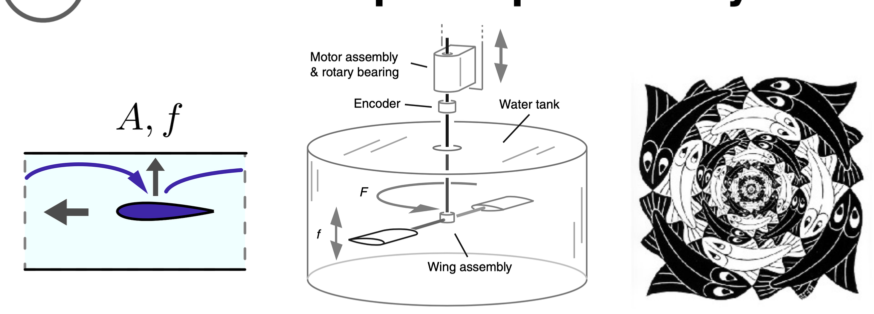


The simulations confirm that the flying formation with multiple flyers is remarkably well-ordered. Since $S_n \approx 1.2$, the wavelength λ traced out by each flyer serves as the appropriate lattice parameter for the crystalline formation.

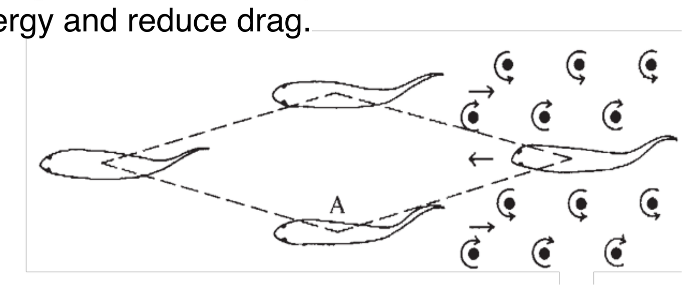


A two-flyer system interacting through the wake emanated from the leader is analogous to a driven, damped harmonic oscillator and this can explain the amplified fluctuations in a resonance cascade.

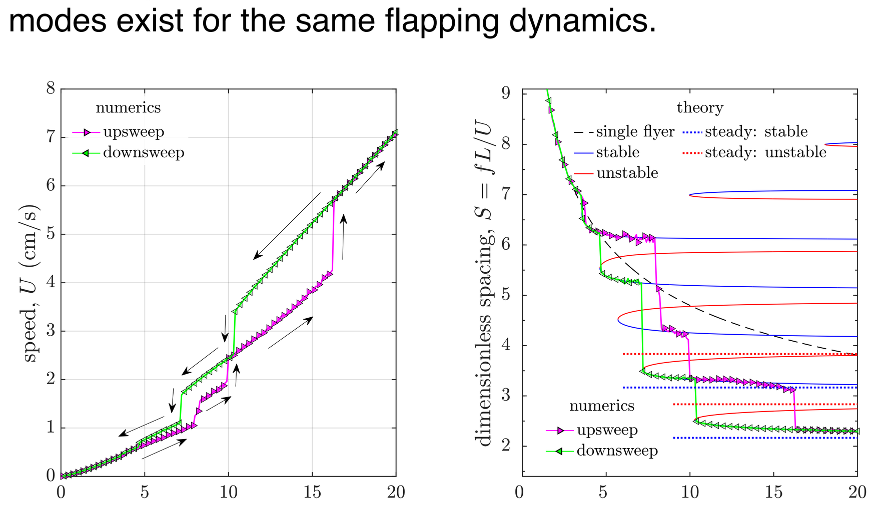
1 Enforced spatial periodicity



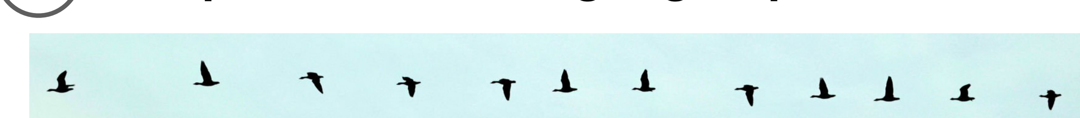
The first setup we investigate is a pair of flyers that are rigidly connected and that are swimming in rotational orbits. This mimics an infinite array of flyers in which the inter-flyer spacing is fixed. This setup bears similarities to Weihs-type models, where the individuals have fixed arrangements and they interact with vortices to save energy and reduce drag.



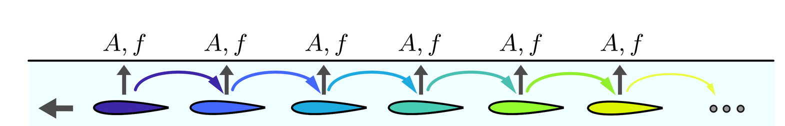
To characterize the motion of the flyer pair we vary the flapping frequency (while keeping the amplitude fixed) and measure the resulting swimming speed around the circular domain. An upward sweep followed by a downward sweep reveals hysteresis loops that show that both fast and slow modes exist for the same flapping dynamics.



5 Implications for larger groups

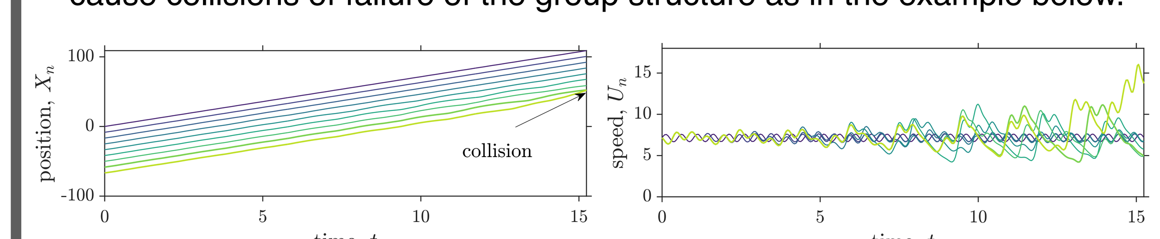


Open boundary conditions

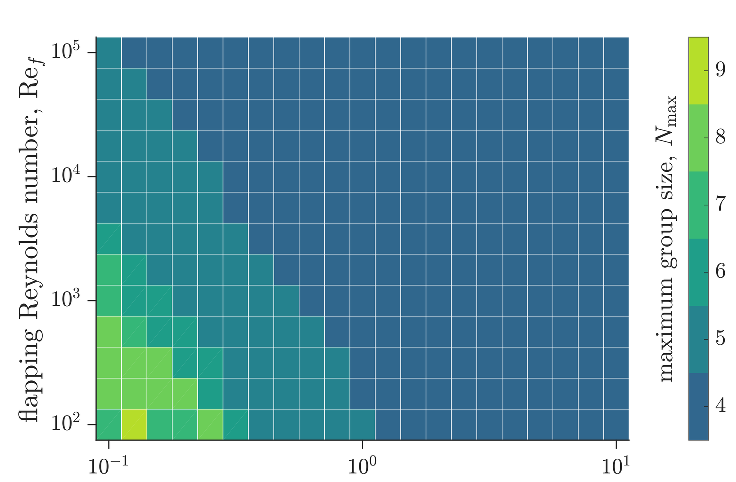


What group sizes are "safe"?

In the few-flyer system we have seen that even in the absence of external perturbations, small disturbances in the group can excite flonons. These excitations are sufficient to trigger instabilities that can cause collisions of failure of the group structure as in the example below.

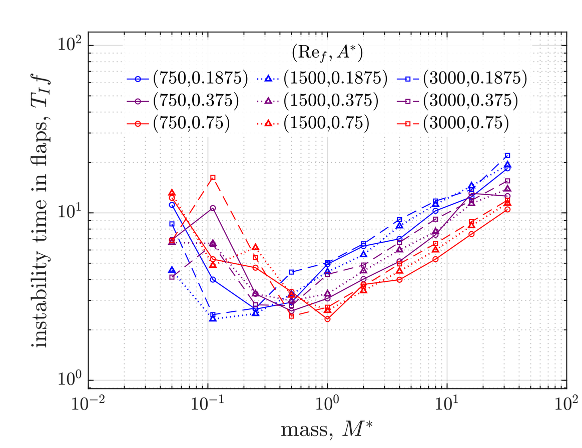


This tendency of disturbances to resonantly amplify severely limit the group size. The figure below shows the maximum group size that survives an initial transient perturbation in the absence of any imposed perturbations in the parameter space of dimensionless mass and flapping Reynolds number. For this choice of dimensionless parameters, the maximum group size that maintains the crystalline structure is between 4 and 9. Therefore, the crystal is extremely brittle.



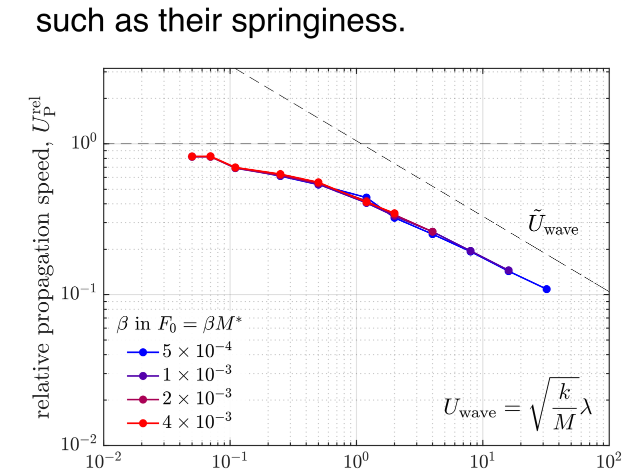
Timescale of instability

The instability grows exponentially. This means that we can estimate the growth rate using the slope of the curve $\log|\Delta U_N|$. The instability timescale is then computed as the reciprocal of the growth rate. We plot the instability time in flaps vs mass and find that the flyers must react very quickly. Without any active control they will collide with their neighbors.



Disturbance propagation speed

Another intriguing analogy with the spring-mass system is the propagation speed of a disturbance as it passes down the group. This disturbance propagation speed is determined by two competing effects: the delay between disturbances left by one member and picked up by the downstream member, and the material properties of the flow interactions such as their springiness.



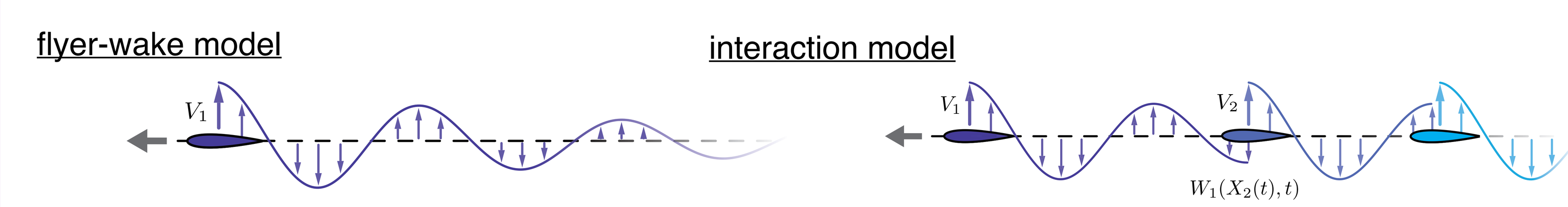
In the small-mass limit, the delay effect dominates and the wave speed is set by the flight speed.

In the large-mass limit, the usual spring-mass wave speed (without delay) limits how fast disturbances travel.

Therefore, the group behaves as an excitable "crystal" with regularly ordered member "atoms" whose positioning is susceptible to deformations and dynamic instabilities.

Follower-wake interaction model

Each flyer has a prescribed flapping and this dictates the flyer's self-propulsion as well as the wake flow signal left behind in its trail. The flyer experiences a propulsive force that depends on how the instantaneous oscillator signal interferes with the ambient wake signal left by others. This addresses the fact that flying formations involve interactions through long-lived flows that have memory of the earlier conditions under which they were generated.



System of delay differential equations

$$\dot{X}_n(t) = U_n(t), \quad \dot{U}_n(t) = \frac{\rho C_T cs}{2M}\left[V_n(t) - V_{n-1}(t_n(t))e^{-(t-t_n(t))/\tau}\right]^2 - \frac{5s\sqrt{\rho\mu c}}{M}U_n^{3/2}(t), \quad \dot{t}_n(t) = \frac{U_n(t)}{U_{n-1}(t_n(t))}$$

Dimensionless version of the model

We non-dimensionalize the system above using the flapping amplitude A, characteristic timescale $1/f$ and the typical flapping speed Af:

$$\tilde{t} = ft, \quad \tilde{X}_n = \frac{X_n}{A}, \quad \tilde{U}_n = \frac{U_n}{Af}$$

The system of delay differential equations simplifies in the no wake-decay limit $\tau \to \infty$:

$$\dot{X}_n(t) = U_n(t), \quad \dot{U}_n(t) = \frac{C_T \pi^2 A^*}{2M^*}\left[\cos(2\pi t) - \cos(2\pi t_n(t))\right]^2 - \frac{5A^*}{M^*}\frac{1}{\sqrt{\mathrm{Re}_f}}U_n^{3/2}(t), \quad \dot{t}_n(t) = \frac{U_n(t)}{U_{n-1}(t_n(t))}$$

The dimensionless groups are $M^* = \frac{M}{\rho c^2 s}$, $\mathrm{Re}_f = \frac{\rho A f c}{\mu}$, $A^* = \frac{A}{c}$, $s^* = \frac{s}{c}$, C_T.

References

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[Emergent spacing] S. Ramananarivo, et al., Phys. Rev. Fluids 1(7), 071201 (2016)
[Kinematic individuality] J. W. Newbolt, J. Zhang & L. Ristroph, PNAS 116(7), 2419-2424 (2019)
[Few-flyer system] J. W. Newbolt, et al., Nat. Commun 15(1), 3462 (2024)
[Implications for larger groups] C. Mavroyiakoumou, J. Wu, L. Ristroph, to be submitted (2024)