Vertical stretches and compressions (sec 6.2)

Thursday, November 5, 2020 6:04 PM

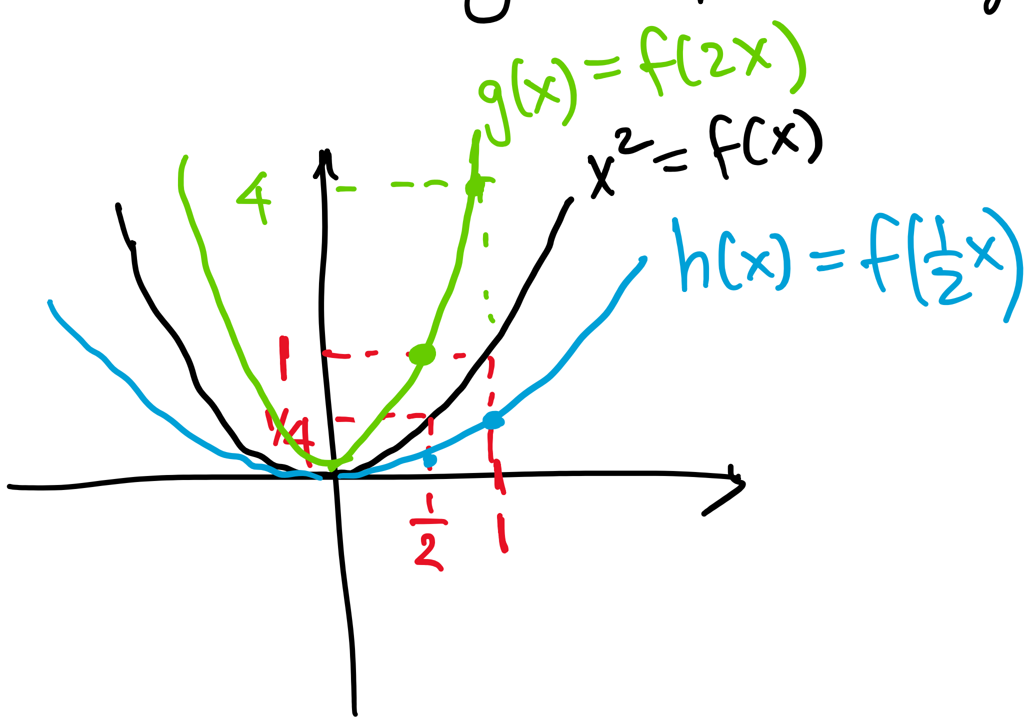
If a function $f(x)$ has the transformation $\boxed{y = k f(x)}$ then

- vertically stretched by a factor k $(k > \overset{1}{\cancel{0}})$
- vertically compressed by a factor of k $(0<k<1)$

If a function $f(x)$ has the transformation $\boxed{y = f(kx)}$ then

- horizontally stretched by a factor of $\frac{1}{k}$ if $0<k<1$
- horizontally compressed by a factor of $\frac{1}{k}$ if $k>1$

e.g.



sketch

$$\boxed{g(x) = f(2x)} = (2x)^2 = \underline{\underline{4x^2}}$$

$x = 1 \Rightarrow g(1) = f(2\cdot 1) = f(2) = 2^2 = 4$

$x = \frac{1}{2} \Rightarrow g\left(\frac{1}{2}\right) = f\left(2\cdot\frac{1}{2}\right) = f(1) = 1$

$$h(x) = f\left(\frac{1}{2}x\right)$$

$x = 1 \Rightarrow h(1) = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$x = \frac{1}{2} \Rightarrow h\left(\frac{1}{2}\right) = f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}.$

General transformations

For any constants A, B, h and k that non-zero the graph of

factored form

$$\boxed{y = A f(B(x-h)) + k.}$$

is obtained by applying the transformations to the graph of $f(x)$ in the following <u>order</u>

★ reflections → stretches / compressions → shifts (do this last)

- If $A<0$ then you have a reflection about the x-axis
- Vertical stretch / compression by a factor of $|A|$ (absolute value)
- Vertical shift of k units

} vertical direction

- If $B<0$ then you have a reflection about the y-axis
- Horizontal stretch / compression by a factor of $\frac{1}{|B|}$ ← abs. value
- Horizontal shift of h units