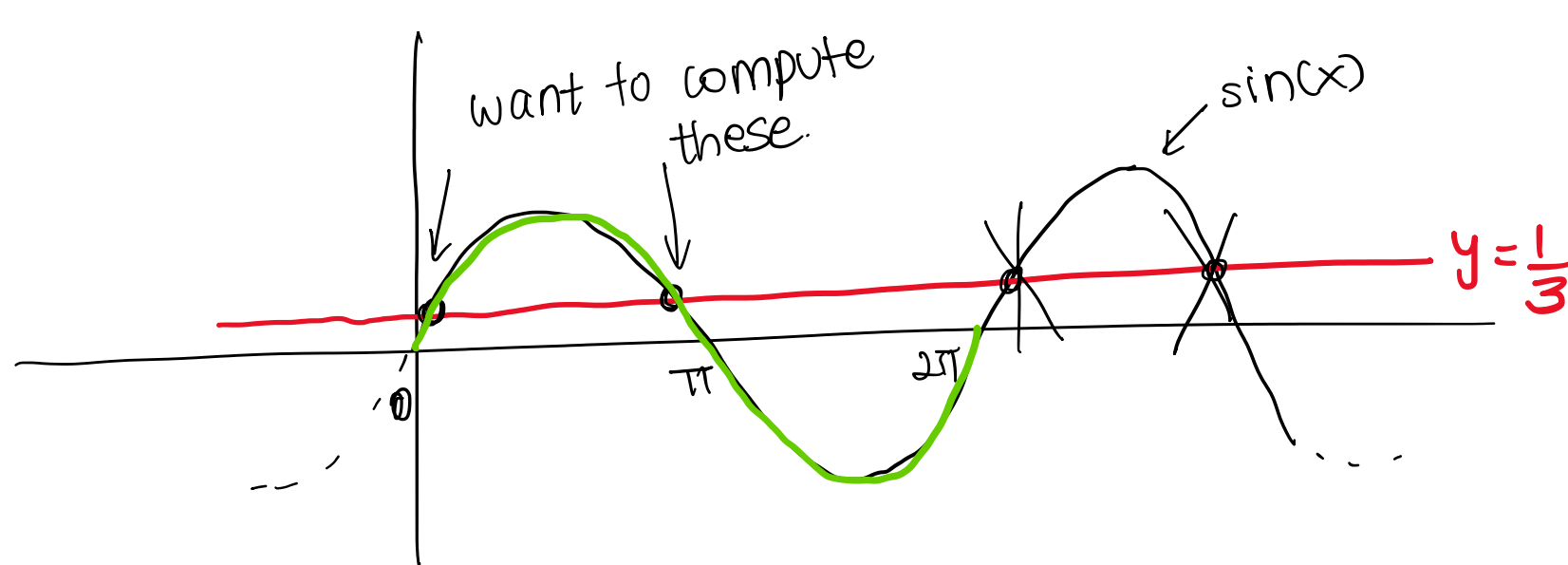# Trigonometric equations (sec. 9.1)

These exercises involve usually finding intersection points between a horizontal line and a trigonometric (sinusoidal) function.

e.g. $\boxed{\sin(x) = \dfrac{1}{3}}$. Solve for $x$ in the interval $0 \le x \le 2\pi$.



__Step 1__: Find first solution by taking $\sin^{-1}$ of the RHS.

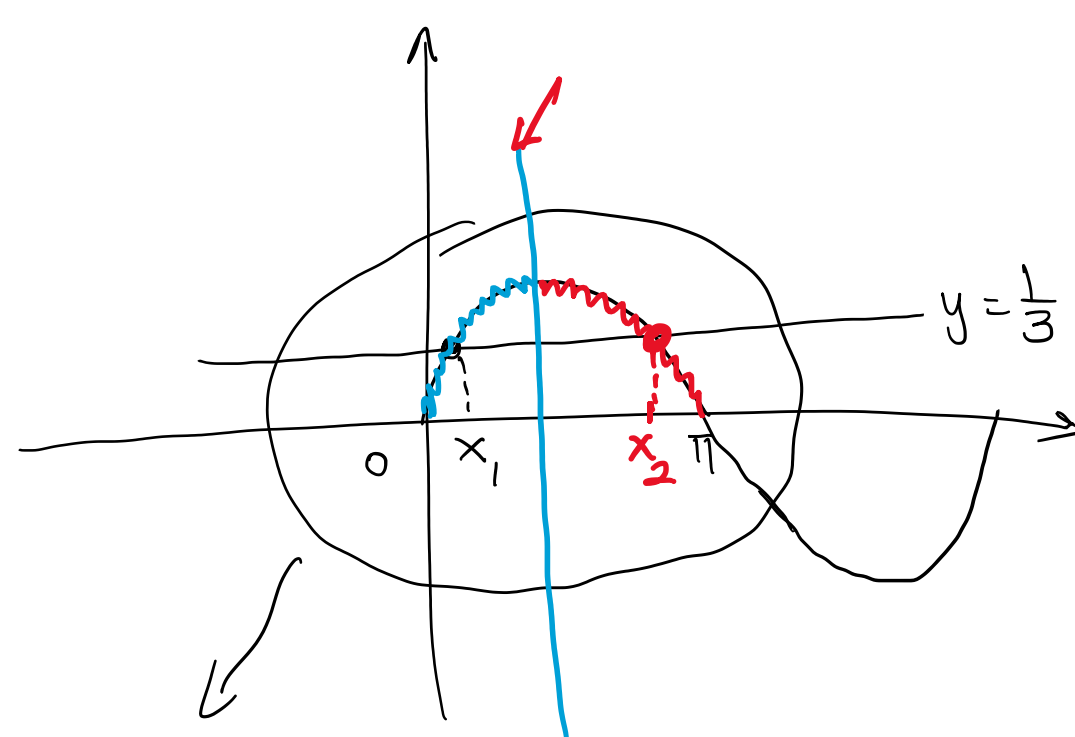$$x_1 = \sin^{-1}\left(\frac{1}{3}\right)$$

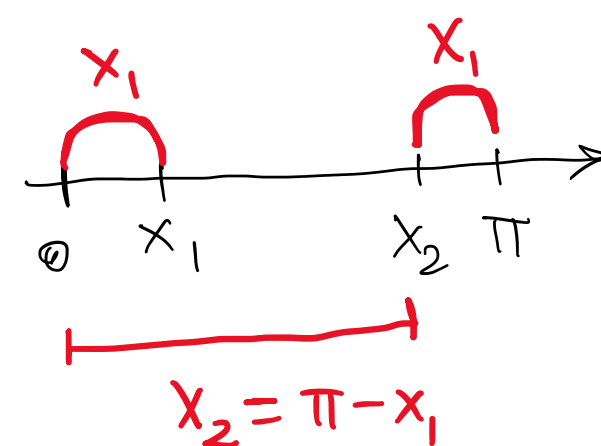__Step 2__ Find the second solution by subtracting the 1st solution from $\pi$.

$$\boxed{x_2 = \pi - x_1}$$
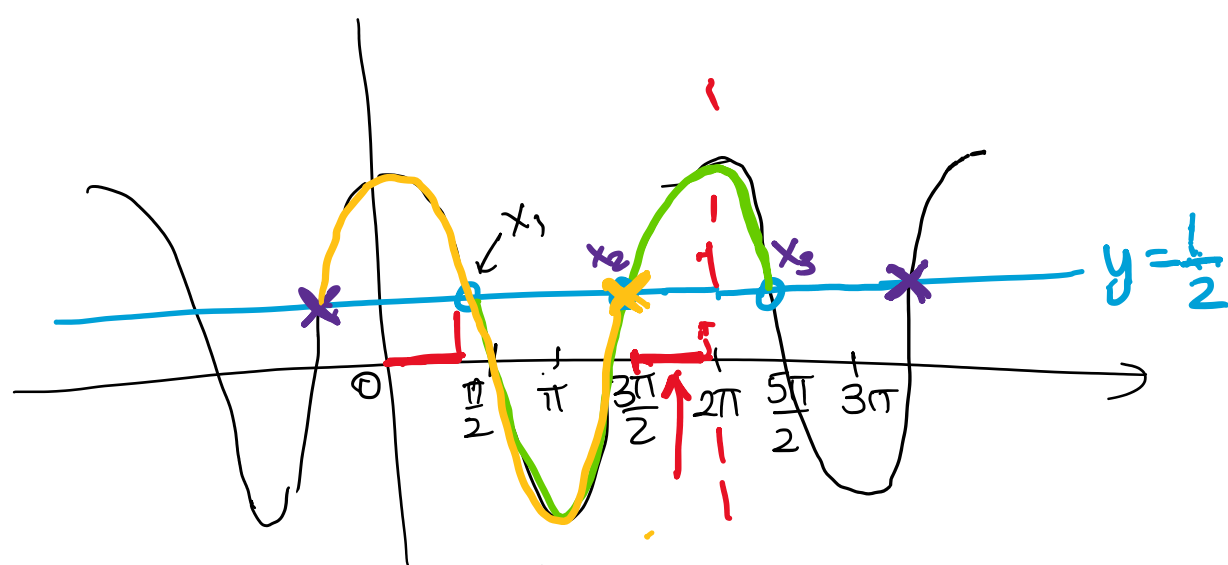
$$x_2 = \pi - \sin^{-1}\left(\frac{1}{3}\right)$$

__Step 3__ To find any other solutions you can add/subtract a whole period $(2\pi)$



zoom-in



$$x_2 = \pi - x_1$$

e.g. $\cos(x) = \dfrac{1}{2}$. Solve for $x$ in the interval $0 \le x \le 3\pi$



__Step 1__   $x_1 = \cos^{-1}\left(\frac{1}{2}\right)$

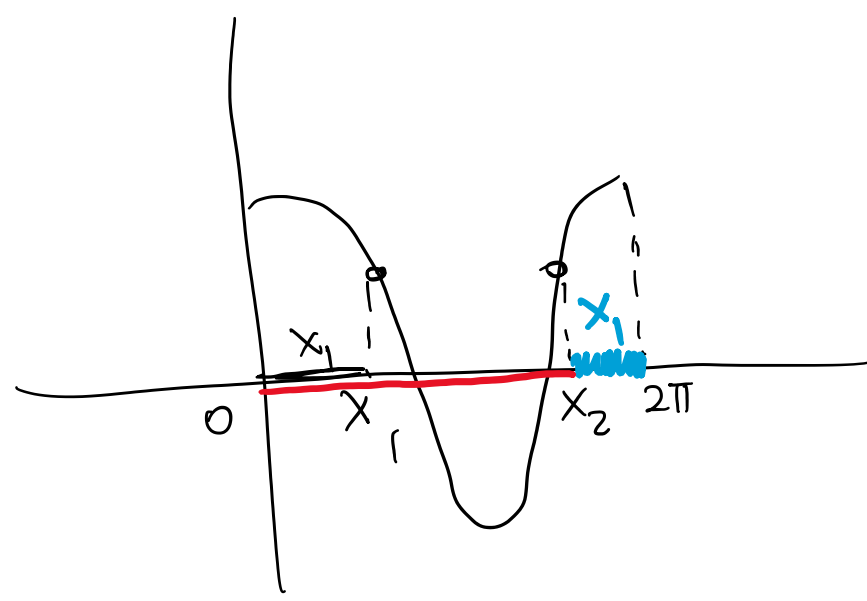__Step 2__ To find the second solution subtract the first from $2\pi$

$$\boxed{x_2 = 2\pi - x_1}$$

$$x_2 = 2\pi - \cos^{-1}\left(\frac{1}{2}\right)$$

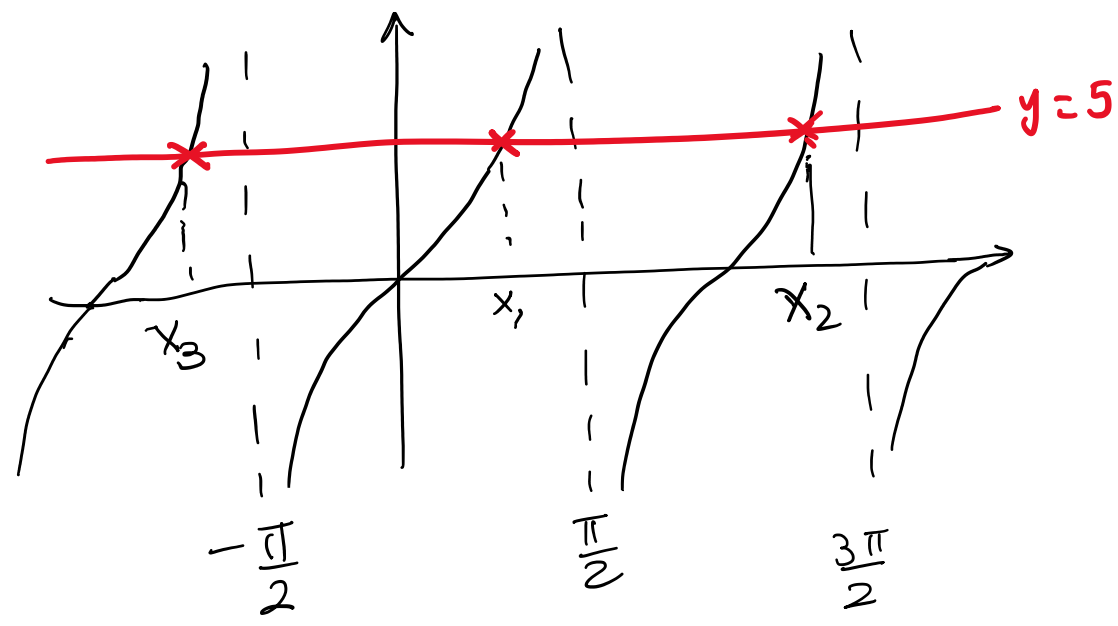__Step 3__ add a period $(2\pi)$ to the 1st solution $\boxed{x_3 = 2\pi + x_1}$

$$x_3 = 2\pi + \cos^{-1}\left(\frac{1}{2}\right)$$



e.g. Solve $\tan(x) = 5$     period $= \pi$



$x_1 = \tan^{-1}(5)$

$x_2 = \pi + x_1 \Rightarrow x_2 = \pi + \tan^{-1}(5)$

$x_3 = x_1 - \pi \Rightarrow x_3 = \tan^{-1}(5) - \pi$

etc.

(add or subtract a period of $\pi$ from subsequent solutions)

# Examples

① Solve $\sin\left(\dfrac{2\pi}{3}(t-5)\right) = \dfrac{1}{4}$ in the interval $0 \le t \le 2\pi$

__Step 1__   $\dfrac{2\pi}{3}(t-5) = \sin^{-1}\left(\frac{1}{4}\right)$   (☺)

__Step 2__   $\dfrac{2\pi}{3}(t-5) = \pi - \sin^{-1}\left(\frac{1}{4}\right)$   (☺)

__Step 3__   Solve all equations for $t$

in (☺):  $t - 5 = \dfrac{3}{2\pi}\sin^{-1}\left(\frac{1}{4}\right) \Rightarrow \boxed{t_1 = \dfrac{3}{2\pi}\sin^{-1}\left(\frac{1}{4}\right) + 5}$

in (☺):  $t - 5 = \dfrac{3}{2\pi}\left(\pi - \sin^{-1}\left(\frac{1}{4}\right)\right) \Rightarrow \boxed{t_2 = \dfrac{3}{2\pi}\left(\pi - \sin^{-1}\left(\frac{1}{4}\right)\right) + 5}$

don't forget the parentheses.

To find any other solutions. (say in the positive $t$-axis)



$$\dfrac{2\pi}{3}(t-5) = \sin^{-1}\left(\frac{1}{4}\right) + 2\pi$$

↑ period

Rearrange for $t$:

$$t - 5 = \dfrac{3}{2\pi}\left(\sin^{-1}\left(\frac{1}{4}\right) + 2\pi\right) \Rightarrow \boxed{t_3 = \dfrac{3}{2\pi}\left(\sin^{-1}\left(\frac{1}{4}\right) + 2\pi\right) + 5}$$