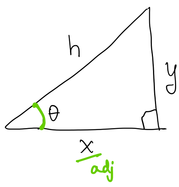
Trig functions with radians (sec 8.1)

Tuesday, November 17, 2020 12:35 AM

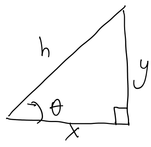
Sine and cosine using right-angled triangles



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{h}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{h}$$

The tangent function

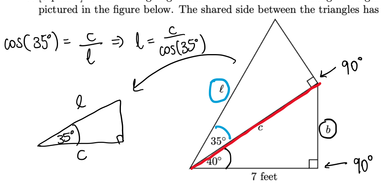


$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

If two side lengths are given and you are trying to find the third side, use Pythagoras' theorem

$$x^2 + y^2 = h^2$$

7. [8 points] Kiki is designing a sail for her new sailboat using two right triangles arranged as pictured in the figure below. The shared side between the triangles has length c.



$\cos(35°) = \frac{c}{\ell} \Rightarrow \ell = \frac{c}{\cos(35°)}$

$\cos(40°) = \frac{7}{c}$

$c = \frac{7}{\cos(40°)}$

Help Kiki by finding the lengths of b, c, and ℓ in **exact** form. None of your answers should include the letters b, c, or ℓ.

a. [2 points] $b =$ $7\tan(40°)$ do not evaluate (exact form)

$\tan(40°) = \frac{b}{7} \Rightarrow b = 7\tan(40°)$

b. [3 points] $c =$ $7\sqrt{1+\tan^2(40°)} = \frac{7}{\cos(40°)}$

Pythagoras' theorem:

$c^2 = 7^2 + b^2$

$= 49 + (7\tan(40°))^2$

$c^2 = 49 + 49\tan^2(40°)$

$\rightarrow c = \sqrt{49+49\tan^2(40°)}$

$= \sqrt{49(1+\tan^2(40°))}$

$= 7\sqrt{1+\tan^2(40°)}$

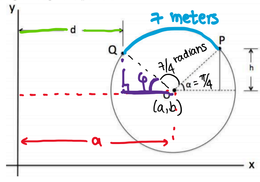
c. [3 points] $\ell =$ $\frac{(7/\cos(40°))}{\cos(35°)}$

8. [5 points] Suppose θ is an angle given in radians with $0 < \theta < \frac{\pi}{2}$ and with $\cos(\theta) = \frac{1}{3}$. Find the following in **exact** form (none of your answers should include the letter θ):

(i) $\sin(\theta) =$ _____

(ii) $\cos(\pi - \theta) =$ _____

7. [10 points] Let C be a circle lying entirely in the first quadrant with radius 4 meters and center at the point $O = (a, b)$ (see the diagram below). A spider is standing at the point P on the circle. The point P makes an angle $\alpha = \frac{\pi}{4}$ radians (measured counterclockwise) with the horizontal line passing through the point O.



$\sin\left(\frac{\pi}{4}\right) = \frac{h}{4}$

$h = 4\sin\left(\frac{\pi}{4}\right)$

a. [2 points] Find the length of the vertical distance h from the point P to the horizontal line passing through the center O of the circle.

$h =$ $4\sin\left(\frac{\pi}{4}\right) = 4\left(\frac{1}{\sqrt{2}}\right)$

b. [3 points] The spider walks 7 meters around the circle, in the counterclockwise direction, from the point P until it reaches the point Q. Find the measure of the angle POQ (in radians).

arclength $= r\theta$ where θ is in radians

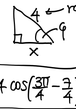
$7 = 4\theta$

$\theta = \frac{7}{4}$ radians

Angle $POQ =$ $\frac{7}{4}$ radians.

c. [5 points] Find the horizontal distance d, in meters, between the point Q and the y-axis. Your answer must be in **exact form** and may contain the constants a and/or b.

$\varphi = \pi - \left(\frac{7}{4} + \frac{\pi}{4}\right) = \frac{3\pi}{4} - \frac{7}{4}$



$\cos\varphi = \frac{x}{4}$

$x = 4\cos\varphi$

$x = 4\cos\left(\frac{3\pi}{4} - \frac{7}{4}\right)$

Finally to find d, $d = a - x \Rightarrow$ $d = a - 4\cos\left(\frac{3\pi}{4} - \frac{7}{4}\right)$

$d =$ _____