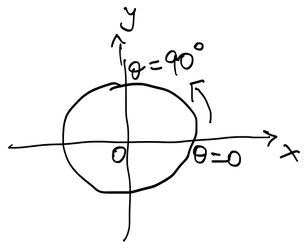
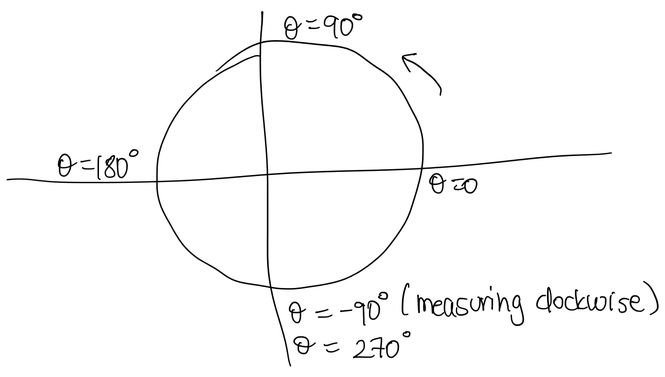
The sine and cosine functions (sec 7.2)

Thursday, November 12, 2020 4:51 PM

Measuring position on a circle using angles.

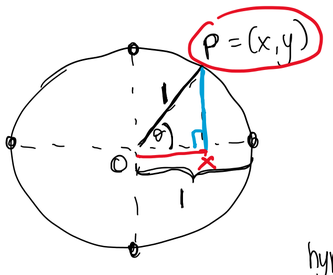
- The circles lies on a coordinate plane with its center at the origin (we'll cover shifted circles later)
- Angles are measured with respect to the positive x-axis
- Positive angles are measured counterclockwise
  (negative angles are measured clockwise)





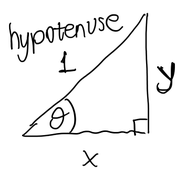
The unit circle

Identity $\cos^2 x + \sin^2 x = 1$



$x = \cos\theta$
$y = \sin\theta$

where $r=1$ (unit circle)



SOH
→ CAH
TOA

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin\theta = \frac{y}{1}$

$\Rightarrow y = \sin\theta$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{1}$

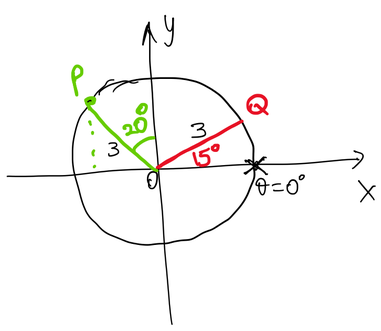
$\Rightarrow x = \cos\theta$

For a circle of radius r we can find the coordinates as

$x = r\cos\theta$
$y = r\sin\theta$

Example

radius = 3



Q: $x = 3\cos(15°) = 2.898$
$y = 3\sin(15°) = 0.776$

P: $x = 3\cos(110°) \simeq -1.026$
$y = 3\sin(110°) \simeq 2.819$

(1 point) **Michigan/precalc/5e/Chap7Sec2/Q23.pg**

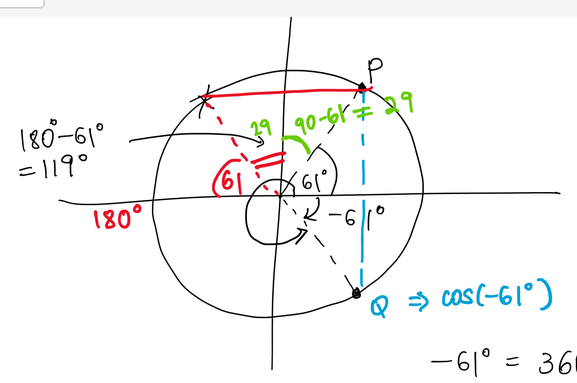
(a) Find another angle ϕ between $0°$ and $360°$ that has the same cosine as $61°$. (That is, find ϕ satisfying $\cos(\phi) = \cos(61°)$.)

$\phi =$ 299° degrees. help (numbers)

x-value

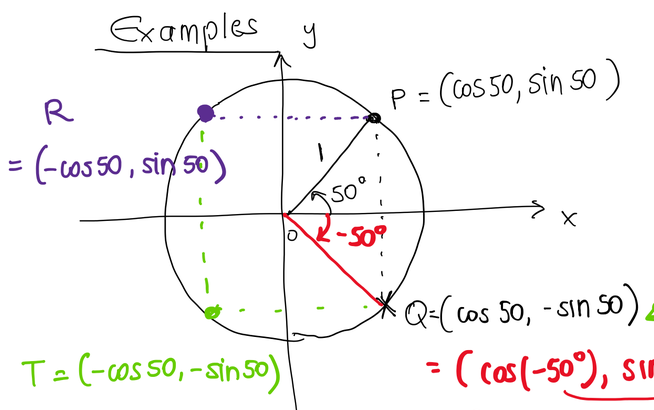
(b) Find another angle ϕ between $0°$ and $360°$ that has the same sine as $61°$. (That is, find ϕ satisfying $\sin(\phi) = \sin(61°)$.)

$\phi =$ 119° degrees. help (numbers)



$-61° = 360° - 61° = 299°$

Examples



= (cos(−50°), sin(−50))

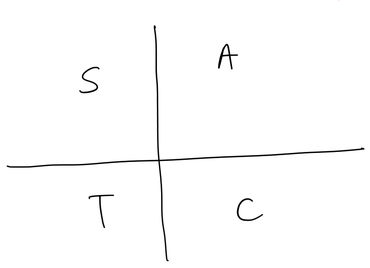
= (cos(50°), −sin(50°))

even
→ $f(x) = f(-x)$

odd
$f(x) = -f(-x)$

Assume cos(x) even
sin(x) odd

$\cos(-x) = \cos(x)$
$\sin(-x) = -\sin(x)$



"All students take calculus"