The short-run behavior of rational function (sec. 11.5)

Monday, December 7, 2020 6:27 PM

The zeros of a rational function

$r(x) = \frac{p(x)}{q(x)}$ where $p(x), q(x)$ are polynomials

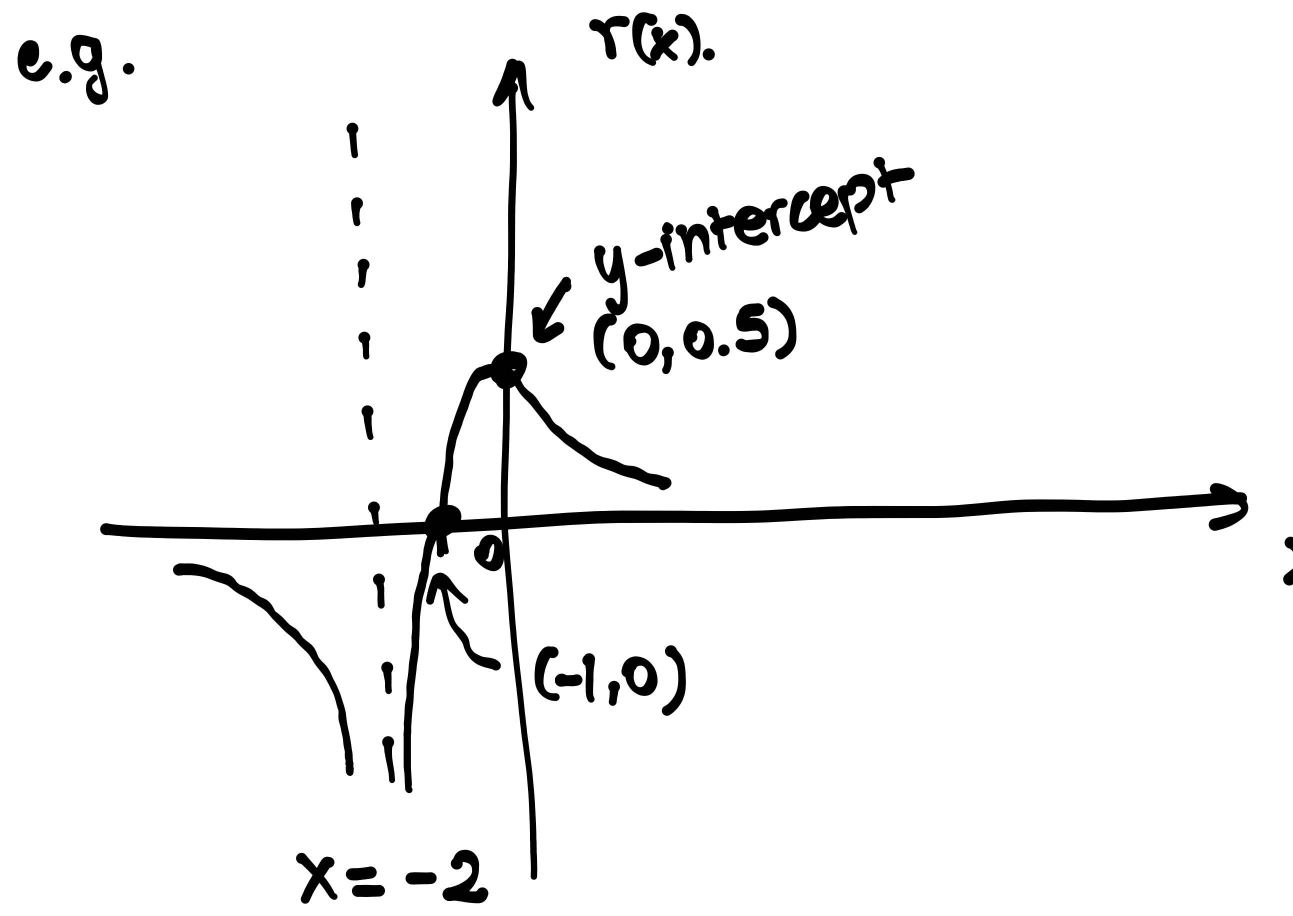
- The zeros of r are the same the zeros of the numerator. Solve $p(x)=0$ for x.

- The vertical asymptote is where the denominator has its zeros. Solve $q(x)=0$ for x.

- Note (from before), the long-run behavior and horizontal asymptote of r is given by the ratio of the leading terms of $p(x)$ and $q(x)$.

Find the formula of a rational function

e.g.



zeros: $x = -1$

vertical asymptote: $x = -2$

$\boxed{\text{y-intercept: } (0, 0.5)}$

unknown, solve for it using the y-intercept.

$r(x) = \frac{k(x+1)}{(x+2)^2} \leftarrow \frac{k(0+1)}{(0+2)^2} = 0.5$

$k\left(\frac{1}{4}\right) = 0.5$

$\boxed{k = 2}$

$\lim_{x \to -2^+} r(x) = \lim_{x \to -2^+} \frac{(x+1)}{(x+2)^2} \to \frac{-1}{(0)^2} \to -\infty$ ✓

but $\lim_{x \to -2^+} r(x) = \lim_{x \to -2^+} \frac{(x+1)}{(x+2)} \to \frac{-1}{-1.9999\ldots+2} \to -\infty$ ✓

$\lim_{x \to -2^-} r(x) = \lim_{x \to -2^-} \frac{(x+1)}{(x+2)} \to \frac{-1}{-2.00001+2} \to \frac{-1}{-0.00001} \to +\infty$

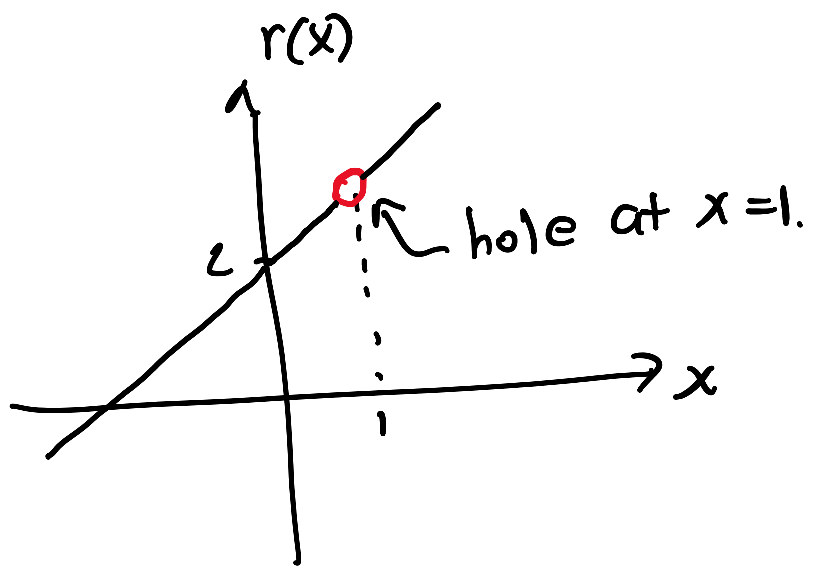
So $\boxed{r(x) = \frac{2(x+1)}{(x+2)^2}}$

Holes of a rational function

When the numerator and denominator have the same zeros.

e.g. $r(x) = \frac{x^2+x-2}{x-1} = \frac{\cancel{(x-1)}(x+2)}{\cancel{x-1}}$

before you simplify you are dividing by zero when $x=1$.



However if you were given $g(x) = \frac{1}{x-1}$