The short-run behavior of polynomials (sec. 11.3)

Thursday, December 3, 2020 4:41 PM

Finding the zeros of a polynomial.

Write the polynomial in factored form with as many linear factors as possible.

Def^n of a zero of a polynomial: The x-values for which $p(x)=0$.

Example $y = x^3 - x^2 - 6x$

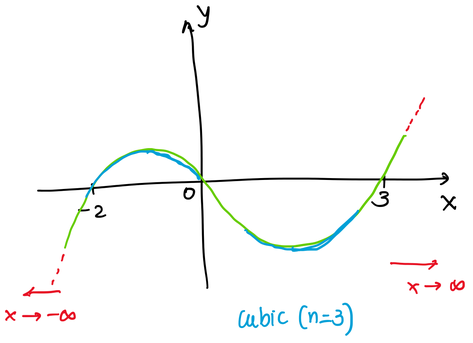
$= x(x^2 - x - 6)$

$= x(x-3)(x+2)$

$= 0 \Rightarrow x=0, x=3, x=-2$

y-intercept: $y=0$ ($x=0$)

x-intercept: $x = -2, 0, 3$ ($y=0$)

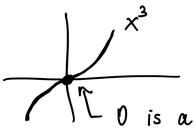


cubic ($n=3$)

Note

- Suppose that p is polynomial. If the formula of p has a linear factor, a factor of the form $(x-k)$ then p has a zero at $x=k$.
- The graph of an n^{th} degree polynomial has at most n zeros and turns at most $(n-1)$ times.

Example



0 is a repeated zero of x^3.

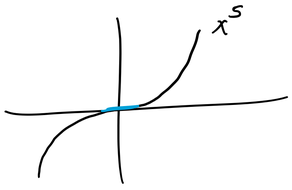
If p is a polynomial with a repeat linear factor $x-k$ then p has a multiple zero.

If $(x-k)$ occurs an even number of times $p(x)$ does not cross the x-axis, it touches it

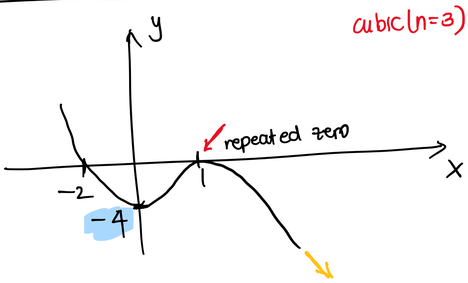
e.g.

If $(x-k)$ occurs an odd number of times it crosses the x-axis and it's flattened there at $x=k$

e.g



Finding formulas for polynomials



cubic ($n=3$)

y-intercept $y=-4$ when $x=0$

solve for this using the y-intercept

$y = k(x-1)^2(x-(-2))$

$y = k(x-1)^2(x+2)$

Use $(0,-4)$ (y-intercept) to find k:

$-4 = k(0-1)^2(0+2)$

$-4 = k(-1)^2(2)$

$-4 = 2k$

$k = -2$

So the formula is $y = -2(x-1)^2(x+2)$

$\rightarrow -2(x)^2(x) = -2x^3$ goes to $-\infty$ as $x \to \infty$