The number e and continuous growth rate (sec 4.5)

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NB. $2 < e < 3$ special number $e \simeq 2.71828\ldots$

Exponential functions with base e:

Any positive base b can be written as a power of e, i.e. $\boxed{b = e^k}$

This implies that $y = ab^t$ can be rewritten in terms of e.

$$Q = ab^t = a(e^k)^t = ae^{kt} \leftarrow$$

If $b > 1$, then $k > 0$ $(b = e^k)$ ← growth

if $0 < b < 1$, then $k < 0$ ← decay

Here k is called the CONTINUOUS growth rate.

Note if a is positive,

- If $k > 0$, Q is increasing
- if $k < 0$, Q is decreasing.

The difference between annual and continuous growth rates.

Q: What's the difference betⁿ a bank account that pays $\underbrace{\text{12\% interest once per year}}_{\text{option 1}}$ and one $\underbrace{\text{that pays 1\% every month}}_{\text{option 2}}$?

A: Assume that you deposit \$1000. Using option 1: $y = 1000(\underbrace{1.12}_{1+0.12})^t$ (t measured in years)

$1+0.12$ ↑ growth rate

After a year $(t=1)$ you have $1000(1.12)^1 = 1120$ dollars.

Option 2: $1000 \xrightarrow{\times 1.01} 1010 \xrightarrow{\times 1.01} 1020.1 \xrightarrow{\times 1.01} \ldots$

after one month

$$= 1000 \underbrace{(1.01)(1.01)\ldots(1.01)}_{12}$$

$$= 1000(1.01)^{12}$$

$$= 1126.83 \text{ dollars}$$