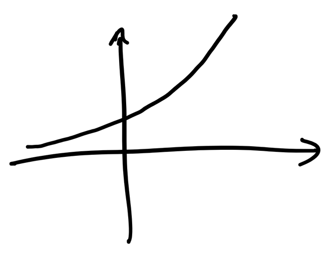
# The logarithmic function and its application (sec 5.3)

Monday, November 2, 2020 5:56 PM

Definition : $y = \log x$ means $10^y = x > 0$

this is always positive

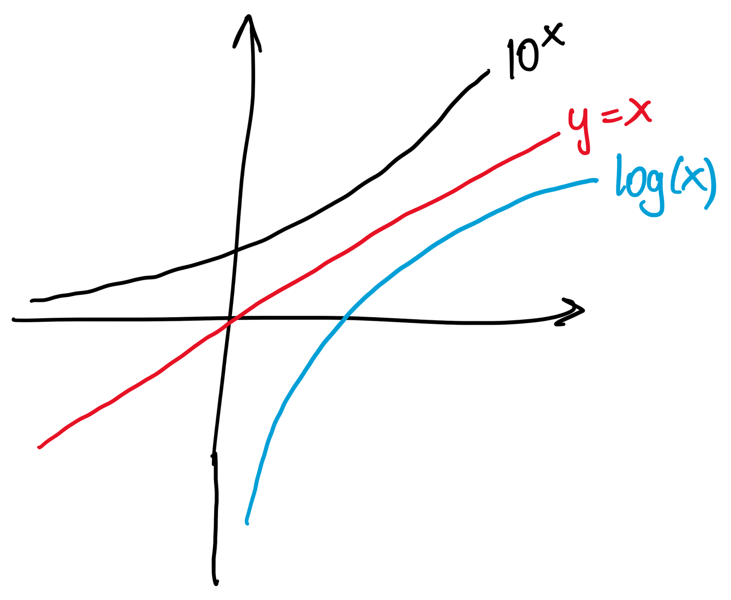


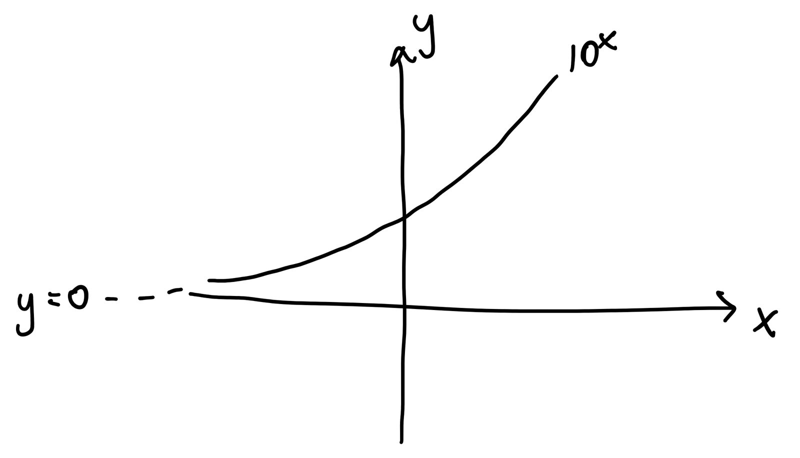
Domain : $x > 0$ (domain of $\log x$ is all positive numbers)

Range : $-\infty < y < \infty$ (range of $\log x$ is all real numbers)

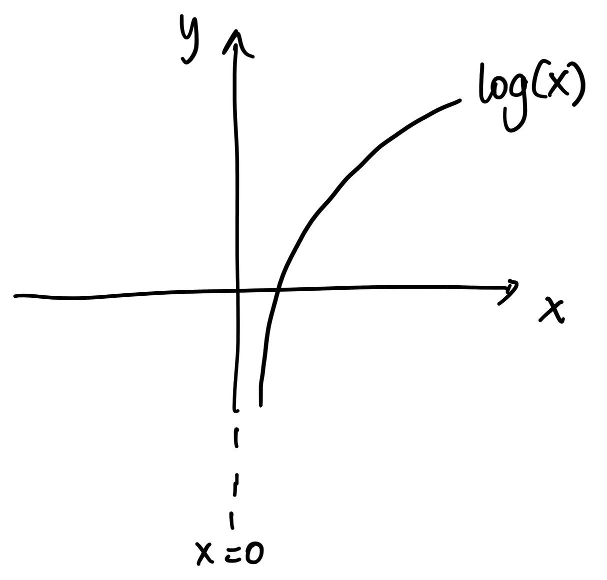
Recall that $10^x$ is the inverse of $\log x$

To plot the graph of $\log(x)$ you can reflect the graph of $10^x$ along the line $y = x$ since they are inverses of each other.




horizontal asymptote for $10^x$ is $y = 0$




vertical asymptote for $\log(x)$ is $x = 0$

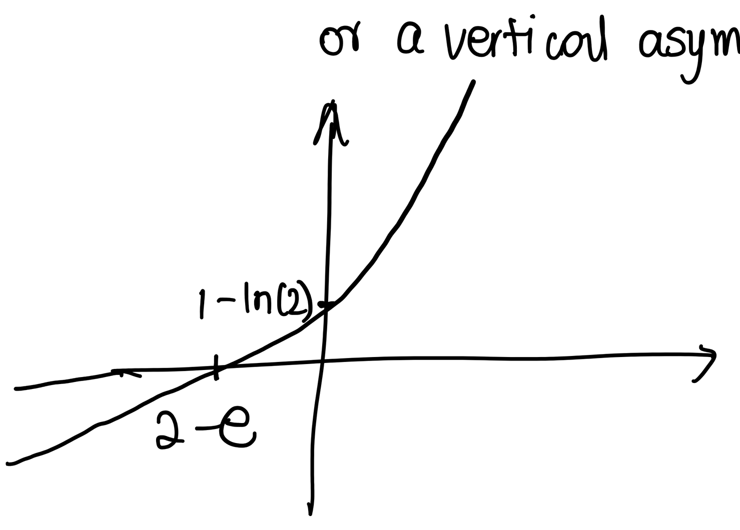
Example Consider $y = 1 - \ln(2-x)$

a) What is the domain? $2 - x > 0$

$x < 2$ $(-\infty, 2)$

$-\infty < x < 2$

b) Determine whether its graph has a horizontal asymptote or a vertical asymptote




y-intercept : $x = 0$

$y = 1 - \ln(2-0)$

$= 1 - \ln(2)$

x-intercept : $y = 0$

$1 - \ln(2-x) = 0$

$-\ln(2-x) = -1$

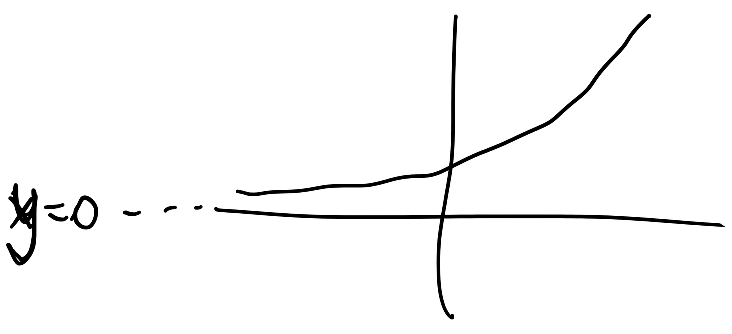
$\ln(2-x) = 1$

$e^{\ln(2-x)} = e^1$

$2 - x = e$

$x = 2 - e$

Reminder: $y = e^x$




$\boxed{\lim_{x \to -\infty} e^x = 0}$

$\lim_{x \to \infty} e^x = \infty$

For horizontal and vertical asymptotes you do the following:

→ let $y = f(x)$ be a function and $a$ be a finite number

The graph of $f(x)$ has a horizontal asymptote of $\boxed{y = a}$ if

as $x \to \infty$ $f(x) \to a$ or as $x \to -\infty$ $f(x) \to a$ or both

→ The graph of $f(x)$ has a vertical asymptote of $\boxed{x = a}$ if

as $x \to a^+$, $f(x) \to \infty$ or as $x \to a^+$, $f(x) \to -\infty$

(approach a from the right)

$\boxed{\text{as } x \to a^-, \ f(x) \to \infty}$ or $x \to a^-$, $f(x) \to -\infty$

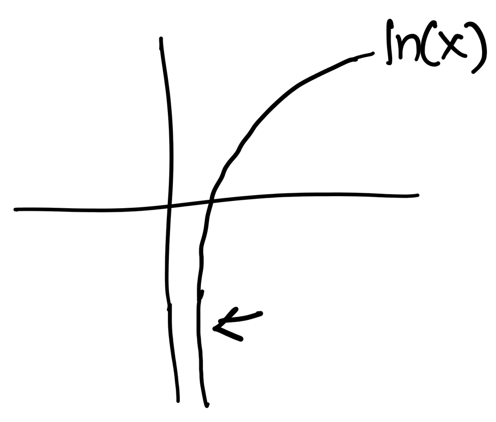
(approach a from the left)

Example $y = 1 - \ln(2-x)$

$\lim_{x \to \infty} 1 - \underbrace{\ln(2-x)}$ not possible

$\lim_{x \to -\infty} \boxed{1 - \ln(\underbrace{2-x}_{\to \infty})} = -\infty$ infinite

no horizontal asymptote




domain $\boxed{x < 2}$ $-\infty < x < 2$

$\lim_{x \to 2^-} 1 - \ln(\underbrace{2-x}_{\to 0}) = 1 - (-\infty) \to \infty$

vertical asymptote $\boxed{x = 2}$

$\lim_{x \to 0^+} \ln(x) = -\infty$