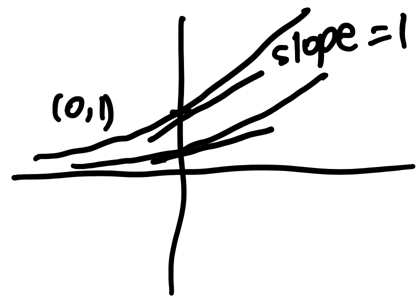
Slope fields

Saturday, July 25, 2020 1:29 PM

Slope fields help us visualize differential equations. Let's take for example

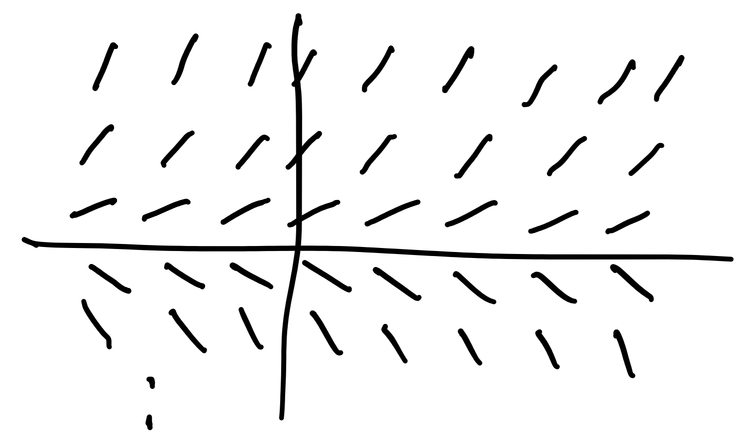
$$\frac{dy}{dx} = y.$$

This implies that any solution to this differential eqn has the property that the slope at any point is the y-coordinate at that point.



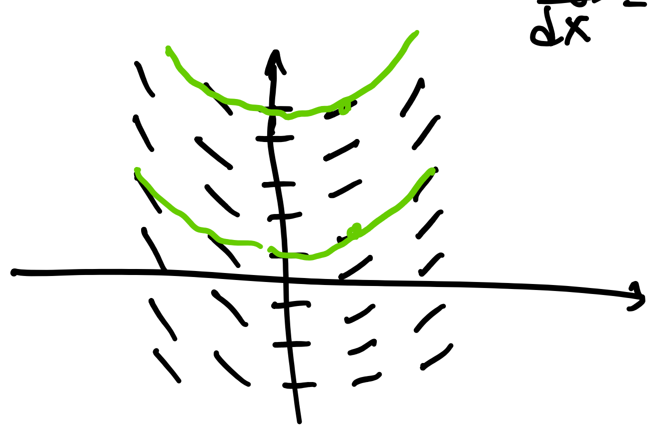
Note slope is constant on a horizontal line where y is constant.

Slope field :



The higher the y-value for $y>0$ then the steeper the slope field line is

How does it behave if $\frac{dy}{dx} = x$?



if you connect the slope fields they should give you a parabola

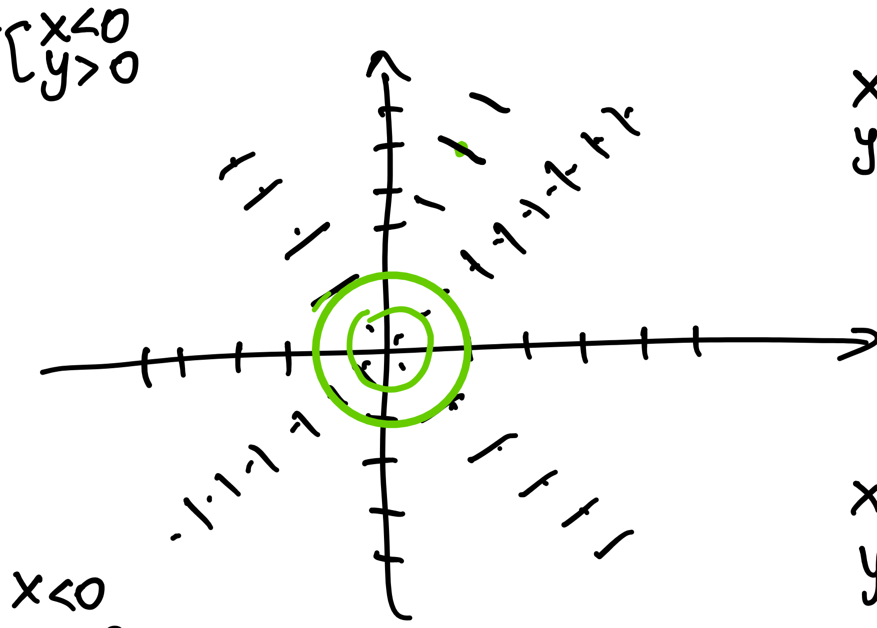
$$\int \frac{dy}{dx}dx = \int x \, dx$$

$$y = \frac{x^2}{2} + C \quad \text{parabolas.}$$

Example $\frac{dy}{dx} = -\frac{x}{y}$

$\frac{dy}{dx} > 0$ ← $\left[\begin{matrix} x<0 \\ y>0 \end{matrix} \right.$

$\left. \begin{matrix} x>0 \\ y>0 \end{matrix} \right] \frac{dy}{dx} < 0$



$\left. \begin{matrix} x>0 \\ y<0 \end{matrix} \right] \frac{dy}{dx} > 0$

$\frac{dy}{dx} < 0$ $\left[\begin{matrix} x<0 \\ y<0 \end{matrix} \right.$

Solution curves: $x^2+y^2 = C$, where C is a constant.

Check using implicit differentiating.

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y} \quad ✓$$